Measurement uncertainty and measurement deviation

Practice

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Definition and Terms

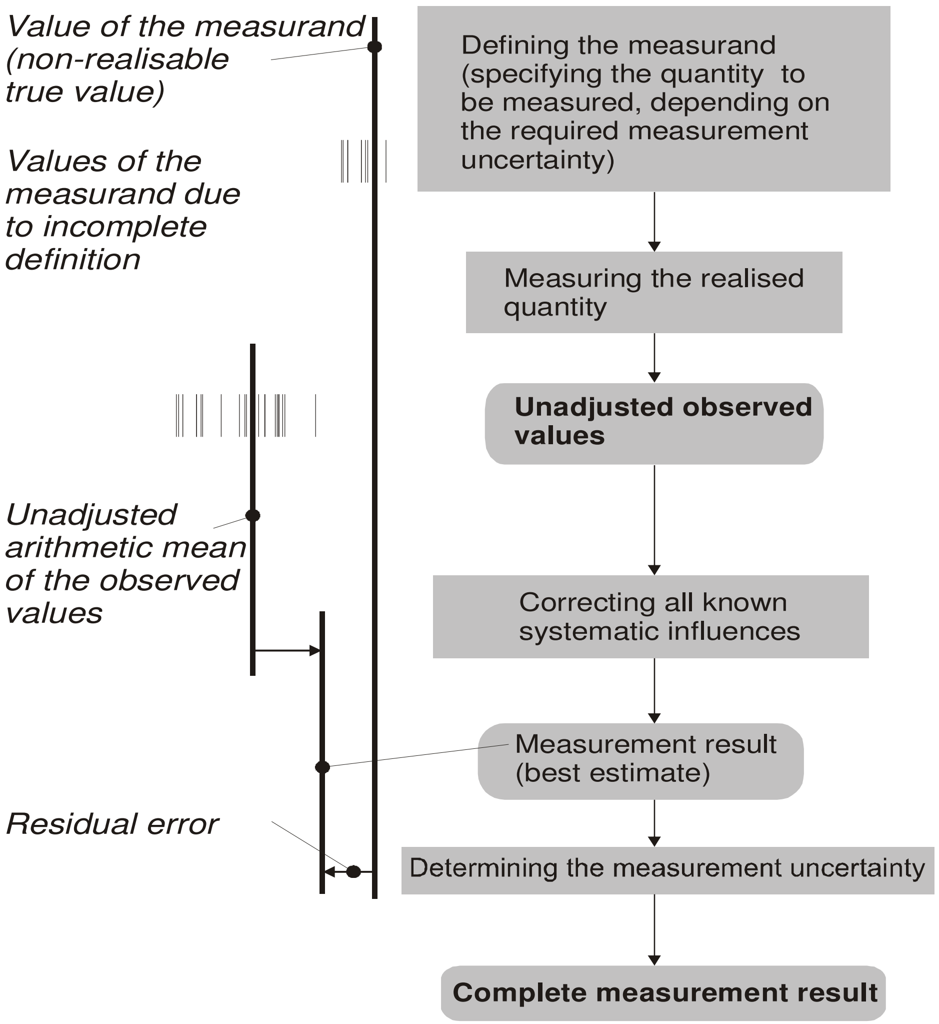
Measurement uncertainty is a parameter associated with the result of a measurement that characterises the dispersion of values which could reasonably be attributed to the measurand.

Mesurement error is the deviation from the true value of a value gained from measurements and assigned to the measurand, or the measurement results minus the true value of the measurand.

The **limit of error** is the maximum amount of measurement deviation of a measuring device.

Influence Factors on Measurement deviation

Value | Proceeding | Example



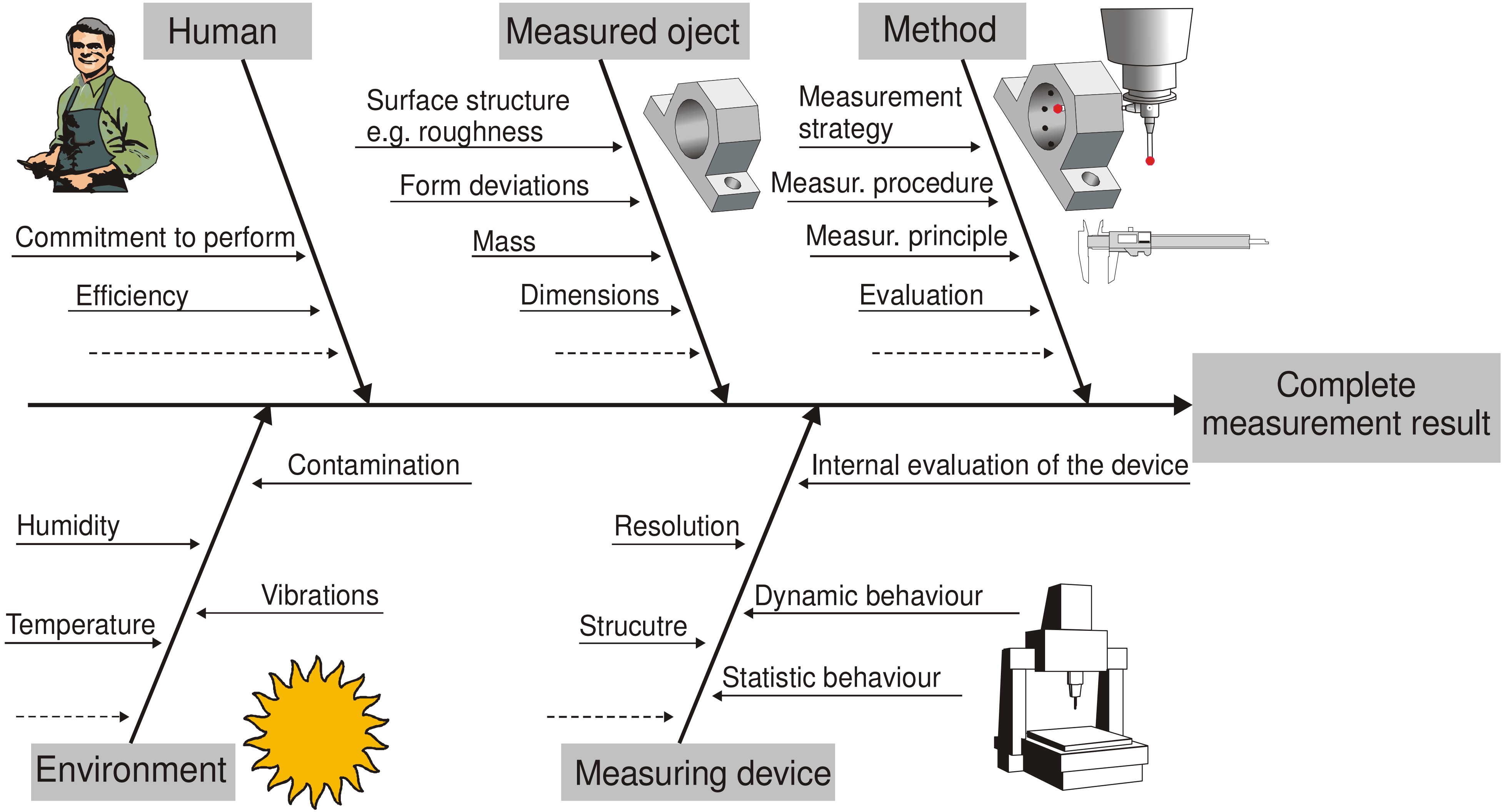
Determining the thickness of a given sheet of material at 20°C

Measuring the thickness of the sheet at 25 C° with a micrometer and measuring the applied pressure

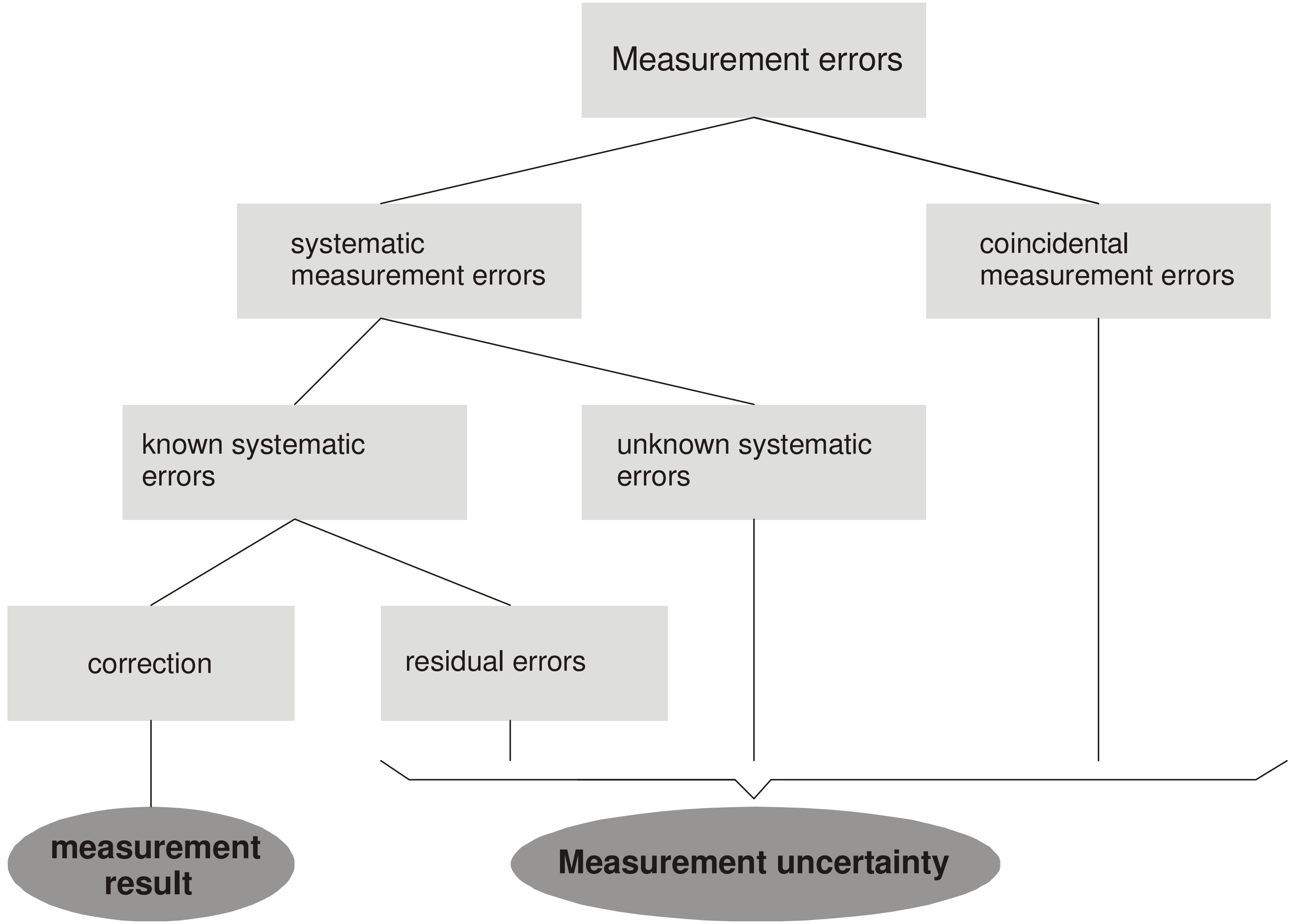
Correcting the influences of temperature and pressure

Taking into account the uncertainty of the micrometer and the correction

Cause and effect diagram of production metrology



Types of measurement errors



Influence Factor Temperature

„Temperature measurement is not everything in length metrology, but it cannot be done without"

Three types of temperature influences:

- Deviation of the temperature level from the reference temperature
- Temporal temperature fluctuations
- Spatial temperature fluctuations

Types of heat transfer:

- Thermal conduction
- Convection
- Heat radiation

Linear expansion characteristic: $D_L = L * \alpha * D_t$

Coefficient of thermal expansion for solid bodies

Aluminium alloy	α = 23..24	[10^{-6}/K]	Steel	a = 10..12	[10-6/K]
Glass	α = 8..10	[10^{-6}/K]	Zerodur	a = 0..0.05	[10-6/K]

Example:

A 100 mm in length Steelruler will stretch by more than 1µm with a temperature difference of 1K!

The effect of change in length are negligible if:

- $\alpha \sim 0$
- $\alpha_{Werk} = \alpha_M$ und $t_{Werk} = t_M$
- $t_{Werk} = t_M = 20$ °C

Coefficient of thermal expansion for solid bodies

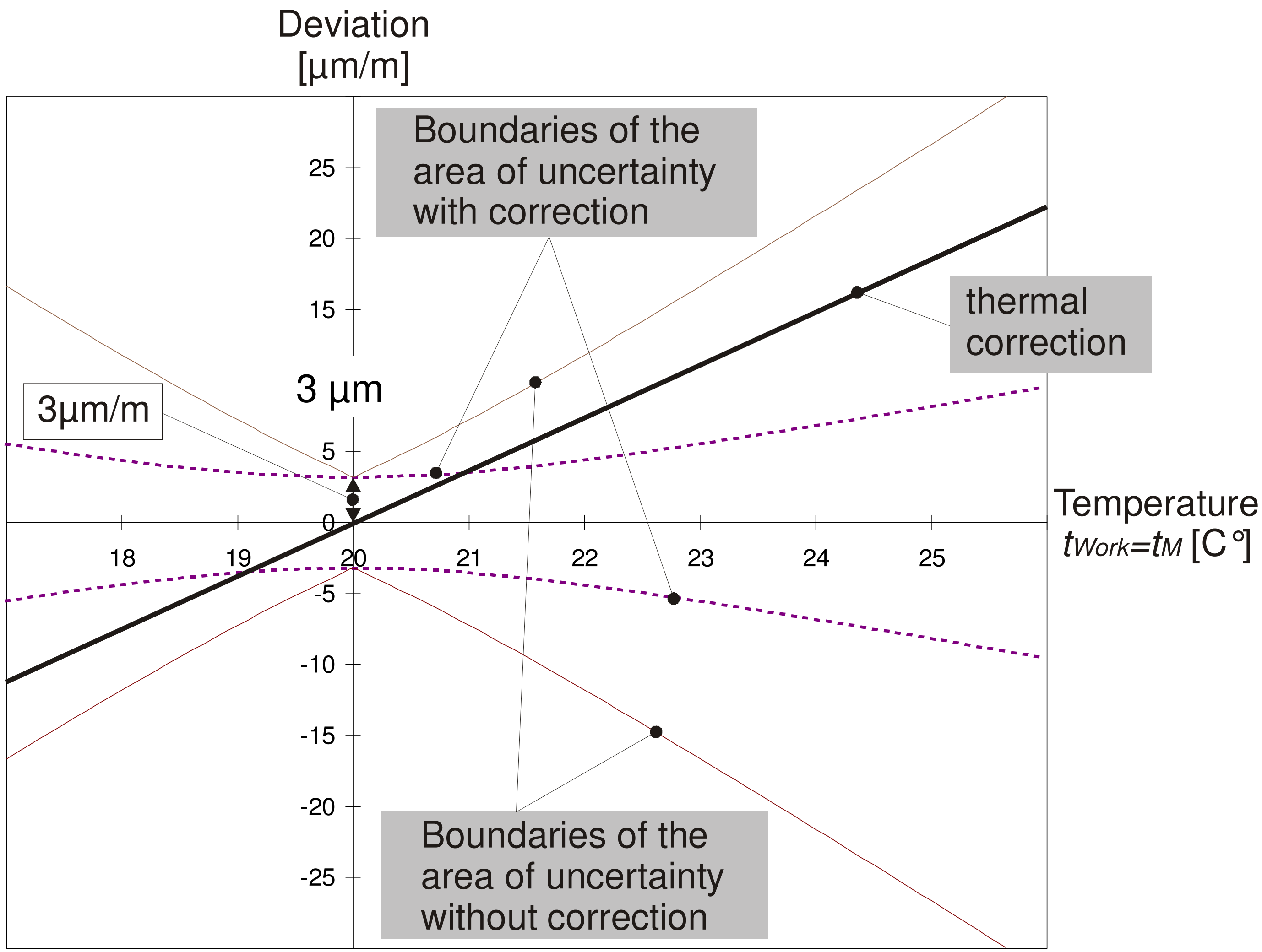
When measuring, the change in length of scale must beconsidered as well as the change in length of the measured objekt:

$$\Delta L = L * (\alpha_{Work} * \Delta t_{Work} - \alpha_M * t_M)$$

The calculable influence of temperature contains an uncertainty which results from the uncertainty of the temperature measurement and the uncertainty of the coefficients of expansion. This is computed by partially deriving and squared addition of the individual parts, where u_a represents the uncertainty of the coefficient of expansion and $u_{\Delta t}$ the uncertainty of temperature measurement:

$$u = L * \sqrt{(u_{\alpha Werk} * \Delta t_{Werk})^2 + (u_{\Delta t Werk} * \alpha_{Werk})^2 + (u_{\alpha M} * \Delta t_M)^2 + (u_{\Delta t M} * \alpha_M)^2}$$

Length measurement error conditional upon temperature

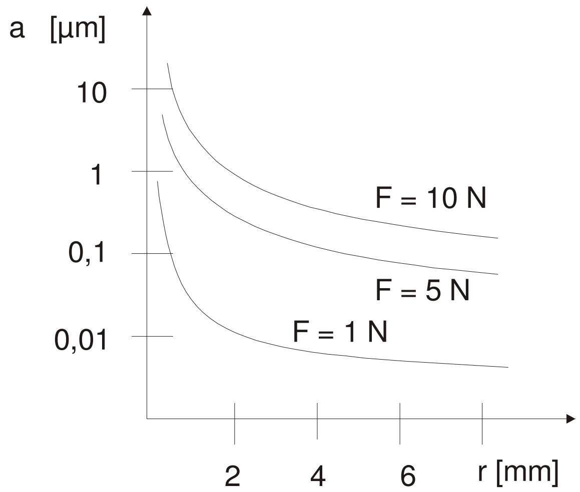


Influence factor: measured object

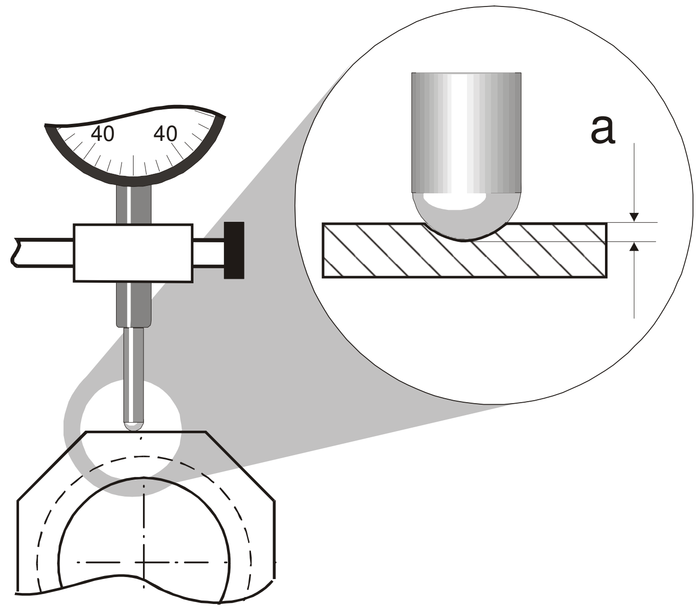
Measurement deviation due to features of the measured object:

- Surface Structure
- Reflection (optical measurement technology)
- Roughness, Waviness (taktile)
- Hardness

Measurement error through deformation

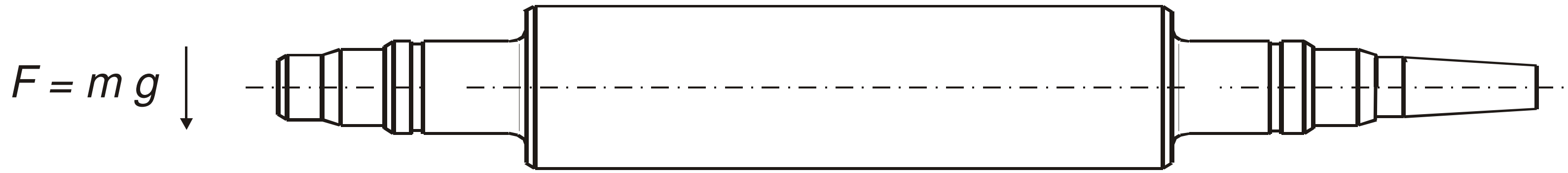


Example: ball-plane (steel)



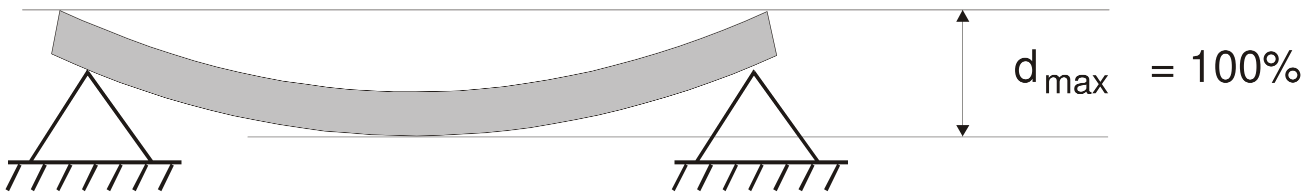
Flattening (Hertz Pressure) through measurement force

Influence factor: slim measured object
e.g. deflection of slim measured objects

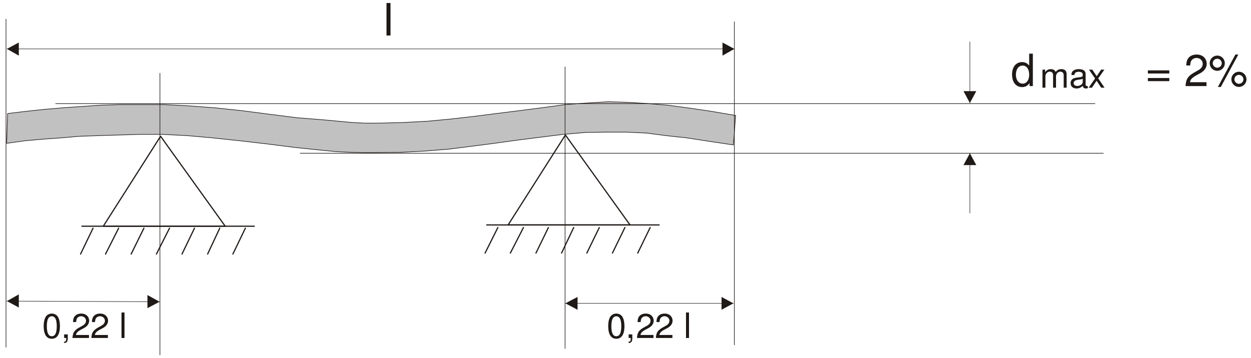


For the deflection of uniform measured objects (pipes, rulers, plates) under dead weight:

a) Support at the ends: maximum deflection



b) Support at each 0,22 l: minimum deflection



Influence Factor: measurement device

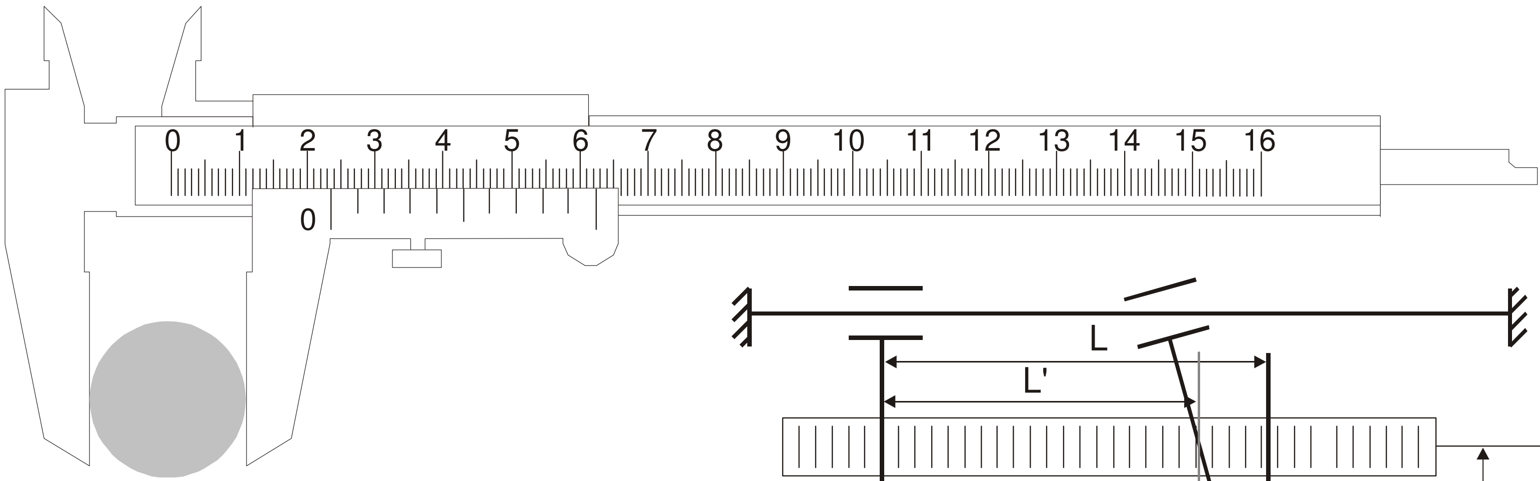
- Measurement errors due to inaccurate guides play an important part in measurement devices with built-in measure embodiment.

- The clearance, which is technically required in guides for measuring pins, touch probes or eyepieces, causes tilting.

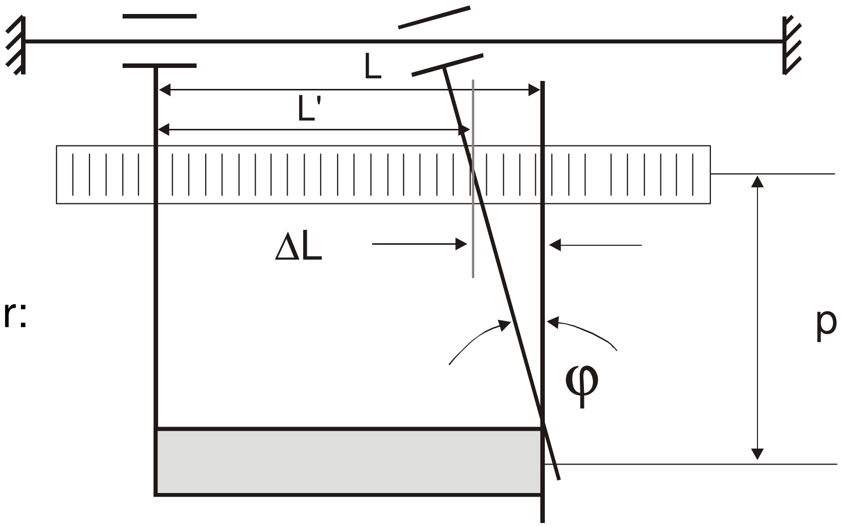
- The influence of these on the measurement result are large (first order) or small (second order), depending on how the measure embodiment and the measured object have been positioned.

➔ Abbe Principle: *In order to avoid errors of the first order, the scale of the measuring device must bepositioned such that the distance to be measured forms a straight-line continuation of the scale.* (Ernst Abbe, 1893)

Influence factor: measurement device – Caliper
Error of first order by Abbe



Measurement Error:
$\Delta L = L' - L \sim p\varphi$



Measured object aligned to measure embodiment
=> complying to the Abbe principle

L' : measured value
L : true value
P : parallel offset

Influence factor: measurement device – Capiler
Breach of Law of Abbe

With a parallel offset of the measuring distance and the reference distance, a small tilt already causes measurement errors which are no longer negligible!

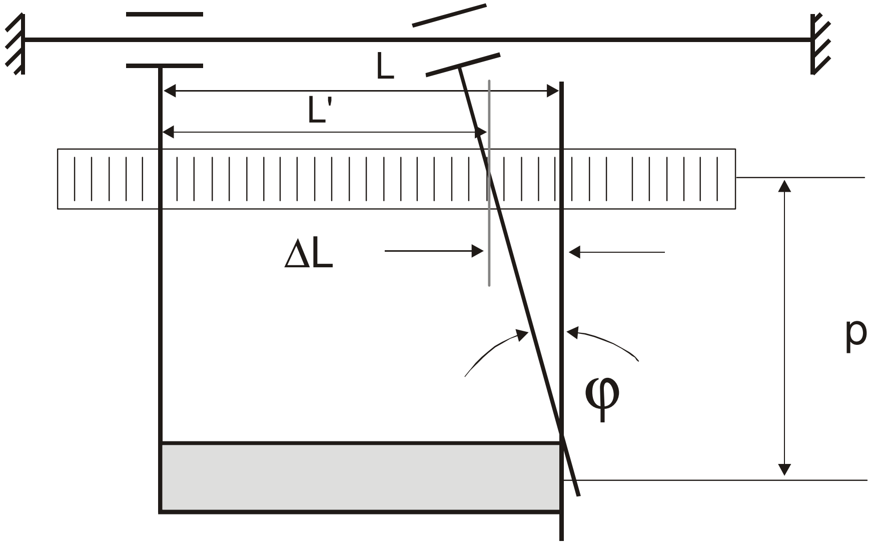
Measurement deviation: $\Delta L = L - L' = p * \tan(\varphi)$

for $\varphi << 1$ with φ described in angle minutes: $\Delta L = p * \varphi$

Example:

- Tilt $\varphi = 2'$
- parallel offset $p = 30$ mm

-> $\Delta L = 17\ \mu m$

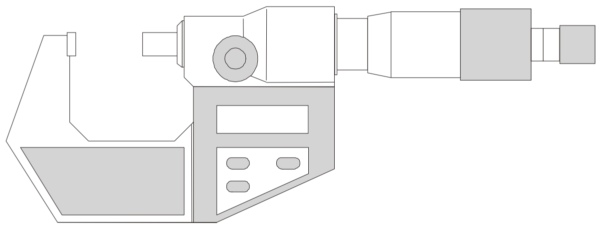


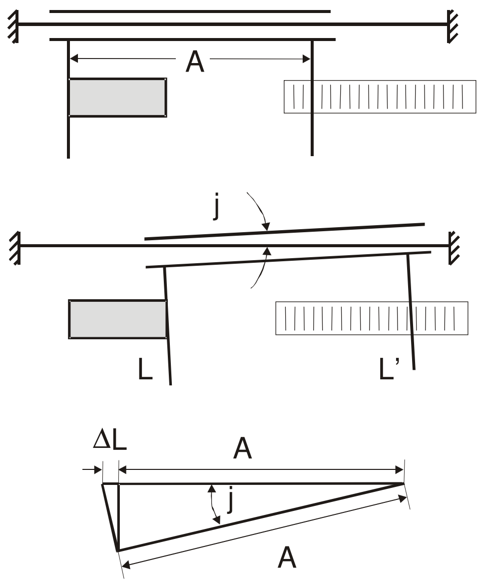
Influence factor: measurement device – micrometer
Abidance of law of Abbe

Measurement object, measurement surface and lead screw form a straight line.

The measurement deviation for φ<<1 can be described as:

$$\Delta L = A\left(\frac{1}{\cos(\varphi)} - 1\right) = A(\sqrt{1+\tan^2(\varphi)} - 1) \approx A(\sqrt{1+\varphi^2} - 1)$$





Measured object aligned to measure embodiment => complying to the Abbe principle

Measurement Error:
$\Delta L = L' - L \sim A^2$

L' : measured value
L : true value

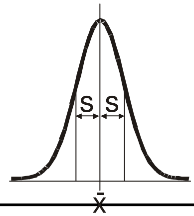
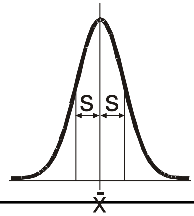
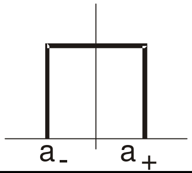
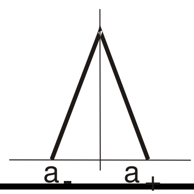
Methods for the estimation of measurement uncertainty

The basis of every procedure for estimating measurement uncertainty ist the "Guide to the Expression of Uncertainty in Measurement" (**GUM)**

The evaluation of a measurement can be carried out in four steps:

i. Setting up a model which mathematically describes the relationships of the measurands (y_1,y_2,...y_n) to all other quantities involved (x_1,x_2,...x_n)
   ➔ $y=f(x_1,x_2,...x_n)$
i. Preparation of the given measurement values and other available data
ii. Calculation of the measurement result and measurment uncertainty of the measurand from the prepared data
iii. Specification of a complete measurement result and determination of the extended uncertainty.

Procedures for estimating measurement uncertainty
ii. Determining the standard uncertainty

Method	Form of distribution	Calculation	
A	Normal distribution 	$u = \frac{s}{\sqrt{n}}$	Standard uncertainty of mean value s: standard deviation n: number of observed values
B	Normal distribution 	$u = \frac{a}{\sqrt{4}}$	Assumption: The estimated value lies within the boundaries a_+ and a_- with a confidence level of 95 %.
	Uniform distribution 	$u = \frac{a}{\sqrt{3}}$	Assumption: The estimated value lies within the boundaries a_+ and a_- with a confidence level of 100 %.
	Tringular distribution 	$u = \frac{a}{\sqrt{6}}$	Assumption: The estimated value lies within the boundaries a_+ and a_- with a confidence level of 100 %.

Procedures for estimating measurement uncertainty
iv. Specification of the complete measurement result

The measurement uncertainty of the measurand, the combined standart uncertainty u_c, is determined through squared addition of the individual uncertainty components. As long as all input variables are independent from each other:

$$u_c = \sqrt{\left(\frac{df}{dx_1}u_{x1}\right) + \left(\frac{df}{dx_2}u_{x2}\right) + \ldots + \left(\frac{df}{dx_n}u_{xn}\right)}$$

or simply:

$$u_c = \sqrt{u^2{}_{x1} + u^2{}_{x2} + \ldots + u^2{}_{xn}}$$

Multiplication with the extension factor k, shows the extended measurement uncertainty:

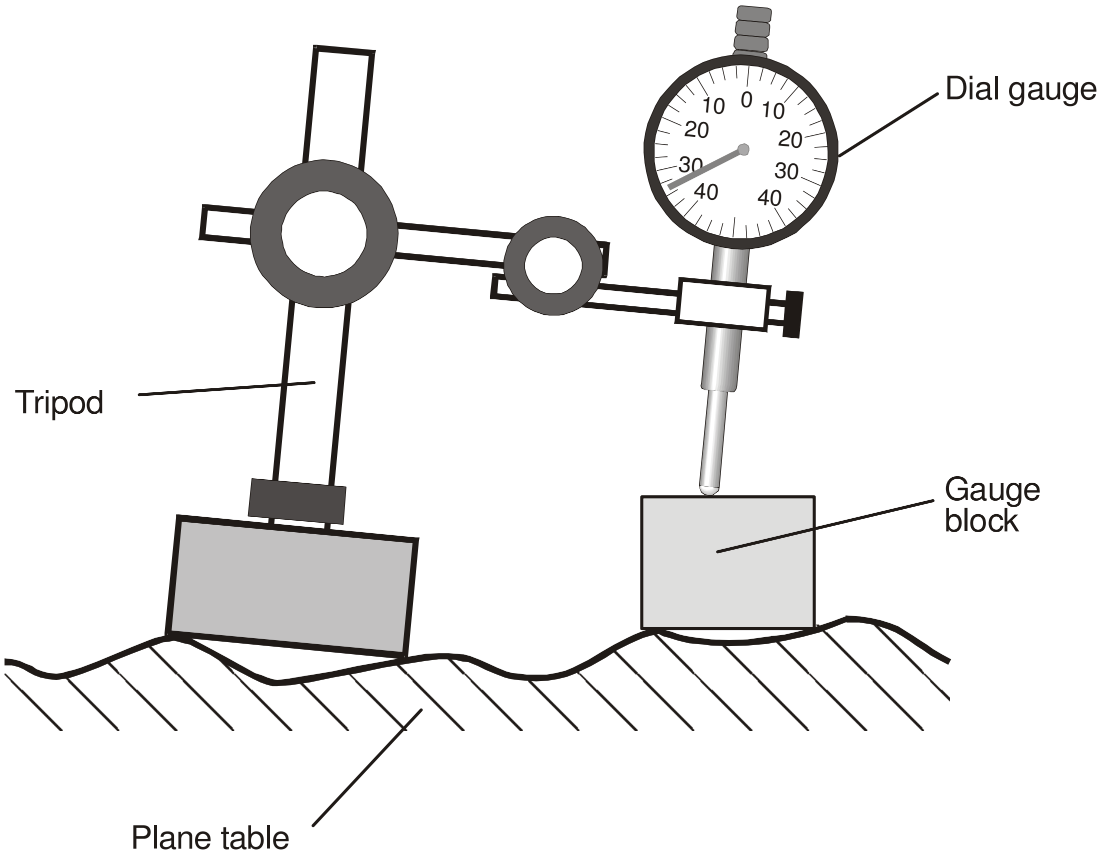
$U = k * u_c$

k = 2 equals a range of confidence of 95%

Evaluation of measurement uncertainty
e.g. Dial Gauge

Example:

Dial Gauge for the determination of the hight of a gauge block



Evaluation of measurement uncertainty
e.g. Dial Gauge

Main influence factors

- Surroundings:
  - The measurement is carried out at 20 °C (error limit of the temperature definition 2 K)
  - Thermal expansion coefficient of the workpiece 12*10-6 /K (uncertainty 1*10-6 /K)
- Measuring device:
  - Measurement deviations for a temperature range of 18-22 °C are in an area of ± 0.02 mm (95% plausibility)
  - Values are distributed normally
  - Systematic deviation of b = -0.06 mm
  - Levelness of the plane table, the support face of the tripod and the formation of the tripod are not known

Evaluation of measurement uncertainty e.g. Dial Gauge - Random influences

20 measurements are taken from different points of the plane table:

- Mean value of the observation: $\bar{x} = 100.02$ mm
- Standard deviation: $s = 0.09$ mm

The systematic deviation necessitates a correction of the mean value for the real value:

$$y = 100.02 \text{ mm} - 0.06 \text{ mm} = 99.96 \text{ mm}$$

Uncertainty components:

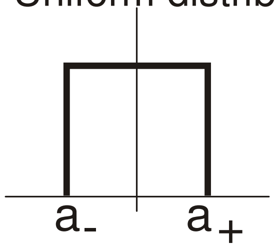
- **Repeated Measurements**

$$u_1 = \frac{0.09 \text{ mm}}{\sqrt{20}} = 0.02 \text{ mm}$$

Evaluation of measurement uncertainty e.g. Dial Gauge – Lenth deviation caused by temperature

- The error limit from the temperature definition is 2K
- When quoting error limits without stating the distribution it is useful to assume a uniform distribution of the values

Uniform distribution



Assumption:
The estimated value lies within the boundaries a_+ and a_- with a confidence level of 100 %.

$$u = \frac{a}{\sqrt{3}}$$

$$u_{\Delta t} = \frac{2\,K}{\sqrt{3}}$$

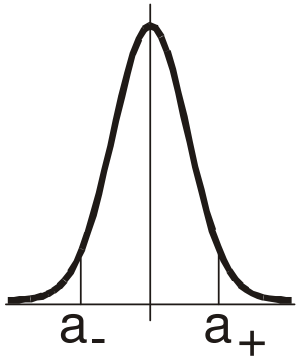
$$u_2 = L * \sqrt{(u_{\alpha Werk} * \Delta t_{Werk})^2 + (u_{\Delta t Werk} * \alpha_{Werk})^2 + (u_{\alpha M} * \Delta t_M)^2 + (u_{\Delta t M} * \alpha_M)^2} \quad \text{(from page 9)}$$

$$u_2 = 99.96\,mm * \sqrt{0 + (\frac{2\,K}{\sqrt{3}} * 12 \bullet 10^{-6}\,\frac{1}{K})^2 + 0 + 0} = 1.4\,\mu m$$

Evaluation of measurement uncertainty e.g. Dial Gauge – Deviation of dial gauge

- normal distribution
- 95% of the values are within a range of ± 0.02mm (Methode B)

Normal distribution



Assumption:
the estimated value lies within the boundaries a_+ and a_- with a confidence level of 95%

$$u = \frac{a}{\sqrt{4}}$$

$$u_3 = \frac{0.02\,\text{mm}}{2} = 0.01\,\text{mm}$$

Evaluation of measurement uncertainty e.g. Dial Gauge – evaluation of the complete measurement result

Combined standard uncertainty:

$$u_c = \sqrt{u_1{}^2 + u_2{}^2 + u_3{}^2} = 0.022\,\text{mm}$$

Extended measurement uncertainty:

➔ extension factor k=2 equals a range of confidence of 95%

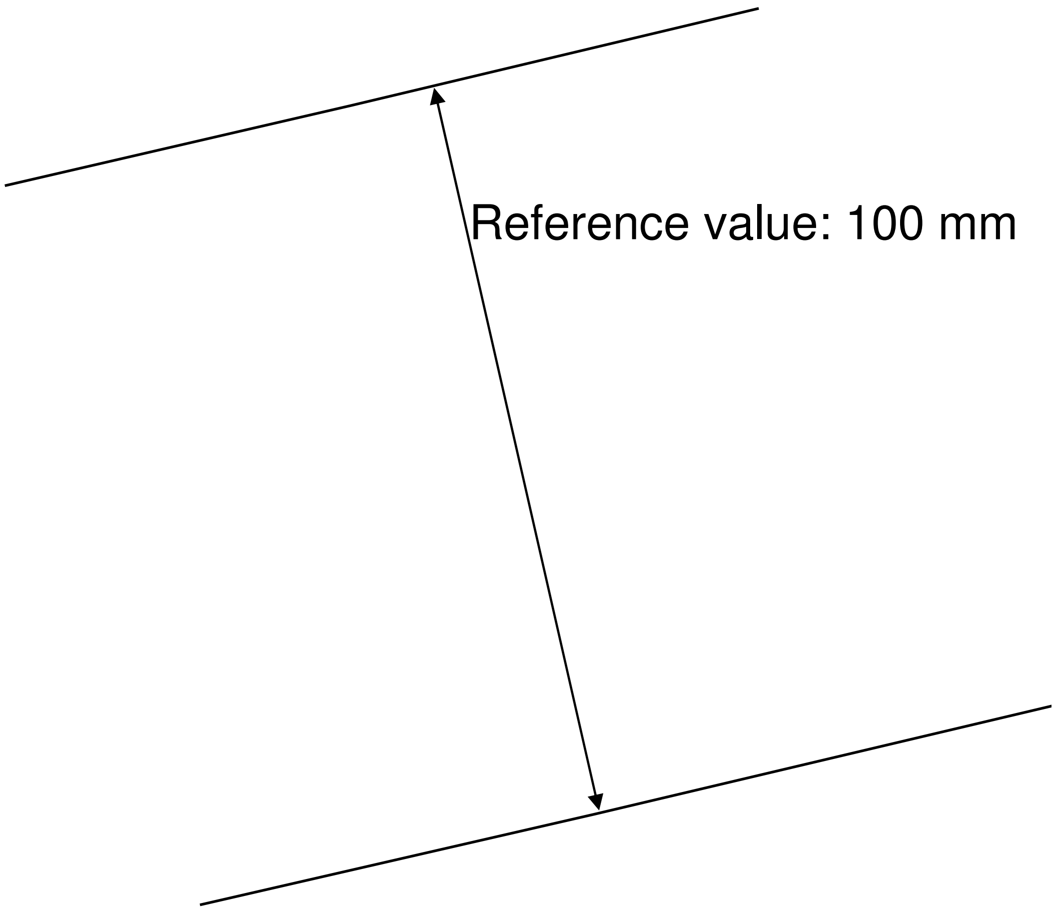
$$U = k^* u_c = 0.044\,\text{mm}$$

Complete result:

$$Y = (\bar{x} + b) \pm U = 99.960\,\text{mm} \pm 0.044\,\text{mm}$$

Practice:
Evaluation of measurement uncertainty for using a ruler

- Take your ruler and measure the distance between the two parallel lines. The nominal size is 100 mm ± 1 mm.
- Estimate the first decimal place (0.1 mm).
- Calculate the mean value and the standard deviation by using the next page.
- Which influences can appear?
- Evaluate the complete measurement result (mean value ± extended value).
- Consider influences of temperature caused length deviation (error limit a = 2K; α Lineal = 120 * 10-6 /K; uniform distributed) and the deviation of the ruler (error limit a = 0.1 mm; nominal distributed).



Practice: series of measurements

Mean value: $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

Standard deviation: $\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2}$

n	x_i	$x_i - \overline{x}$
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
	$\overline{x}$	σ

Influences:

Human:____

Environment:____________________________________

Measured object:_________________________________

Device:____

Übungsaufgabe: Berechnung der Messunsicherheit

Influences	**Error limit or standard deviation**	**Method and distribution**	**calculation**	**Standard uncertainty**
Random variance				
Temperature deviation				
Error of ruler				

Combined standard uncertainty: ______

Extended uncertainty: ______

Complete measurement result: ______