ACCREDITATION SCHEME FOR LABORATORIES

SAC-SINGLAS TECHNICAL GUIDE 3

Guidance on Measurement Uncertainty for Civil Engineering and Mechanical Testing Laboratories

Technical Guide 3, First Edition, Jun 07
The SAC Accreditation Programme is managed by SPRING Singapore

SECTION	CONTENT	PAGE
1.	Introduction	1
2.	Reasons for Estimating Uncertainty	1
3.	General Requirements	1
4.	General Principles	2
5.	Methods of Estimating Measurement Uncertainty	3
6.	Methods of Reporting Test Results	3
7.	Assessment of Compliance with Specification	5
8.	References	7
9.	Acknowledgement	8
A.	GUM Approach - Appendix A	9
	Worked Examples – Based on GUM approach	13
	Example (A1) - Compressive Strength of Hardened Concrete Cubes	14
	Example (A2) - Marshall Stability for Cored Premix Asphalt Sample	20
	Example (A3) - Maximum Density of Gravelly Soils	25
	Example (A4) - Tensile Strength of Metallic Materials	30
	Example (A5) - Density of Hardened Concrete Cube, Measurement Uncertainty Estimated using Numerical Differentiation	39
B.	ISO/TS 21748 Approach - Appendix B	45
	Worked Examples – Based on ISO/TS 21748 Approach	51
	Example (B1) - Compressive Strength of Hardened Concrete Cubes	52
	Example (B2) - Concrete Non-Destructive Testing - Ultrasonic Pulse Velocity Test on Wall & Slab	63
	Example (B3) - Concrete Non-Destructive Testing - Windsor Probe Test on Wall	73
	Example (B4) - Concrete Non-Destructive Testing - Rebound Hammer on Wall & Slab	83

1 INTRODUCTION

1.1 Knowledge of the uncertainty of measurement of testing results is fundamentally important to laboratories, their clients and all institutions using these results for comparative purposes. Uncertainty of measurement is a very important measure of the quality of a result or a testing method. The level of uncertainty that is acceptable has to be decided on the basis of fitness for purpose, the decision having been reached in consultation with the client. Sometimes a large uncertainty may be accepted; at other times a small uncertainty is required.

2 REASONS FOR ESTIMATING UNCERTAINTY

2.1 The uncertainty of a result is a quantitative indication of the quality of the result or the test method.

2.2 The expression of the uncertainty of a result allows comparison of results from different laboratories, or within a laboratory or with reference values given in specifications or standards.

2.3 The uncertainty of measurement may need to be taken into account when interpreting the result under certain circumstances. For example, variation in results from different batches of material will not indicate real differences in properties or performance if the observed differences could simply be accounted for by the uncertainty of the results.

2.4 The uncertainty of measurement is also to be considered when the client or the specification calls for a statement of compliance.

2.5 An understanding of the measurement uncertainty can be a key part in the validation of in-house or laboratory-developed methods. Such methods can be fine-tuned through systematic assessment of the factors influencing the test results based on the understanding of the principles of the method and practical experience of its application.

2.6 An estimation of the components contributing to the overall uncertainty of a test result provides a means of establishing whether the equipment used is capable to provide precise and accurate measurements.

2.7 A consideration of uncertainty components may also indicate that certain aspects of a test method can be improved.

3 GENERAL REQUIREMENTS

3.1 It is SAC-SINGLAS policy to apply the requirements pertaining to the estimation and reporting of measurement uncertainty in accordance to ISO/IEC 17025. There are various published approaches to the estimation of measurement uncertainty in testing. ISO/IEC 17025 does not specify any particular approach. Laboratories are encouraged to use statistically valid approaches. All approaches that give a reasonable estimate and are considered valid within the relevant technical discipline are equally acceptable and no one approach is favoured over the others.

The following are examples of approaches:

(a) Guide to the Expression of Uncertainty in Measurement (GUM) (see clause 5.4.6.3, note 3 of ISO/IEC 17025) is often regarded as having the more rigorous approach to the estimation of uncertainty. However, in certain cases, the validity of results from a particular mathematical model may need to be verified, e.g. through inter-laboratory comparisons.

(b) Both the repeatability and reproducibility (from interlaboratory comparisons) described in ISO 5725 (see clause 5.4.6.3, note 3 of ISO/IEC 17025) may be used in estimating measurement uncertainty according to ISO/TS 21748. However, these may omit some uncertainty sources that should also be estimated and combined, if significant.

(c) In those cases where a well-recognised test method specifies the limits to the values of the major sources of uncertainty of measurement, and specifies the form of presentation of calculated results, the laboratory can be considered to have satisfied the uncertainty of measurement requirements (see clause of 5.4.6.2, note 2 ISO/IEC 17025) by following that test method.

3.2 Until further international development, SAC-SINGLAS will concentrate on the introduction of uncertainty of measurement for quantitative testing results.

3.3 According to ISO/IEC 17025, testing laboratories shall report the estimated uncertainty of measurement, where applicable, under the following circumstances:

(a) when the information on uncertainty is relevant to the validity or application of the tests results

(b) when it is required by the client

(c) when the uncertainty affects compliance to a specification limit i.e. the interpretation of the results could be compromised by a lack of knowledge of the uncertainty (Please refer to Section 7 for guidance on this case)

4 GENERAL PRINCIPLES

4.1 The objective of a measurement is to determine the value of the **measurand**, i.e. the specific quantity subject to measurement. When applied to testing, the general term measurand may cover many different quantities, e.g. the strength of a material, the level of noise measurement and the fire resistance of doors, etc. A measurement begins with an appropriate specification of the measurand, the generic method of measurement and the specific detailed measurement procedure.

4.2 In general, no measurement or test is perfect and the imperfections give rise to **error of measurement** in the result. Consequently, the result of a measurement is only an approximation to the value of the measurand and is only complete when accompanied by a statement of the **uncertainty** of that approximation.

4.3 **Errors** of measurement may have two components, a **random** component and a **systematic** component. Uncertainties arise from random effects and from imperfect correction for systematic effects.

4.4 **Random errors** arise from random variations of the observations (random effects). Every time a measurement is taken under the same conditions, random effects from various sources affect the measured value. A series of measurements produces a scatter around a mean value. A number of sources may contribute to variability each time a measurement is taken, and their influence may be continually changing. They cannot be eliminated but the uncertainty due to their effect may be reduced by increasing the number of observations and applying statistical analysis.

4.5 **Systematic errors** arise from systematic effects, i.e. an effect on a measurement result of a quantity that is not included in the specification of the measurand but influences the result. These remain unchanged when a measurement is repeated under the same conditions, and their effect is to introduce a displacement between the value of the measurand and the experimentally determined mean value. They cannot be eliminated but may be reduced, e.g. a **correction** may be made for the known extent of an error due to a recognised systematic effect. If no corrections are applied to the measurement, the difference between true value and measured value can be considered as an uncertainty component.

5 METHODS OF ESTIMATING MEASUREMENT UNCERTAINTY

5.1 ISO/IEC 17025 does not specify any particular approach to estimate measurement uncertainty. All approaches that give a reasonable estimate and are considered valid within the relevant technical discipline are equally acceptable. The following are examples of possible approaches:

- GUM Approach – Details are as shown in **Appendix A**

- ISO/TS 21748 Approach – Details are as shown in **Appendix B**

6 METHODS OF REPORTING TESTS RESULTS

6.1 The extent of the information given when reporting the result of a test and its uncertainty should be related to the requirements of the client, the specification and the intended use of the result. The following information should be available either in a report or in the records of the test or both:

(a) method used to calculate the uncertainty of the results

(b) list of uncertainty components and documentation to show how these were evaluated, e.g. record of any assumptions made and the sources of data used in the estimation of the components

(c) sufficient documentation of the steps and calculations in the data analysis to enable a verification of the calculation if necessary

(d) all corrections and constants used in the analysis, and their sources.

6.2 When reporting the result and its uncertainty, the use of excessive number of digits should be avoided. It usually suffices to report uncertainty estimates to no more than two significant figures (although at least one more significant figure should be used during the stages of estimation and combination of uncertainty components in order to minimise rounding errors). Similarly, the numerical value of the result should be rounded so that the last significant digit corresponds to the last significant digit of the uncertainty. Further details of significant digits are as shown in **Appendix C.**

6.3 The uncertainty of measurement is obtained by multiplying the combined standard uncertainty by an appropriate coverage factor, k, which is estimated from Student t-distribution table with known degree of freedom and corresponding level of confidence. Further details are given in **Appendix D**.

It is a widely held view that, for most measurement systems, the approximation to a normal distribution of the combined uncertainty is reliable up to two standard deviations, but beyond that the approximation is less reliable. This corresponds to a 95% confidence level.

SAC-SINGLAS has taken a stand to estimate measurement uncertainty to at least 95% confidence level

6.4 The result of the measurement should be reported together with the **expanded uncertainty** and coverage factor appropriate to the **level of confidence** such as in the following example:

Measured Value	78.2 (units)
Expanded Uncertainty	± 0.7 (units)

Or

Measured Value	78.2 (units)
Expanded Relative Uncertainty	± 0.9 (%)

The reported uncertainty is an expanded uncertainty with a coverage factor of k=2, which provides a level of confidence of approximately 95%.

6.5 In some cases, where a particular factor or factors can influence the results and the magnitude cannot be either measured or reasonably assessed, the statement will need to include reference to that fact, for example:

The reported uncertainty is an expanded uncertainty with a coverage factor of k=2, which provides a level of confidence of approximately 95% but excluding the effect of…………………..

6.6 The report shall state whether sampling and/or sub-sampling is carried out by the laboratory. Where sampling / sub-sampling is carried out by the laboratory, the report shall state whether this sampling / sub-sampling uncertainty is included in the expanded uncertainty. For example:

(a) Where a unit sample is delivered by the client to the laboratory a statement such as "**unit samples delivered by the client to laboratory**" and "**sampling uncertainty is not included in the expanded uncertainty**" should be stated in the report.

(b) Where a bulk sample is delivered by the client and sub-sampling is performed by the laboratory, a statement such as "**sub-sampling from a bulk sample of xxx kg given by the client is performed by the laboratory**" and "**sub-sampling uncertainty component is / is not included in the expanded uncertainty**" should be stated in the report.

7 ASSESSMENT OF COMPLIANCE WITH SPECIFICATION

7.1 When the client or the specification requires a statement of compliance, there are a number of possible cases where the uncertainty has a bearing on the compliance statement and these are examined below:

(a) The simplest case is to clearly state that the measured result, extended by the uncertainty at a given level of confidence, shall not fall outside a defined limit or limits. In these cases, assessment of compliance would be straightforward.

(b) The specification requires a compliance statement in the certificate or report but makes no reference to taking into account the effect of uncertainty on the assessment of compliance. In such cases, it may be appropriate for the user to make a judgement of compliance, based on whether the result is within the specified limits, with no account taken of the uncertainty. This is often referred to as 'shared risk', since the end-user takes some of the risk that the product may not meet the specification. In this case there is an implicit assumption that the magnitude of the uncertainty is acceptable and it is important that the laboratory should be in the position to determine the uncertainty.

(c) In the absence of any specified criteria, eg sector-specific guides, test specifications, client's requirements or codes of practice, the following approach is recommended:

i. If the limits are not breached by the measured result, extended by the expanded uncertainty interval at a level of confidence of 95%, then compliance with the specification can be stated (Case A, Fig 1 and Case E, Fig 2)

ii Where an upper specification limit is exceeded by the result even when it is address by half of the expanded uncertainty interval, then non-compliance with the specification can be stated (Case D, Fig 1)

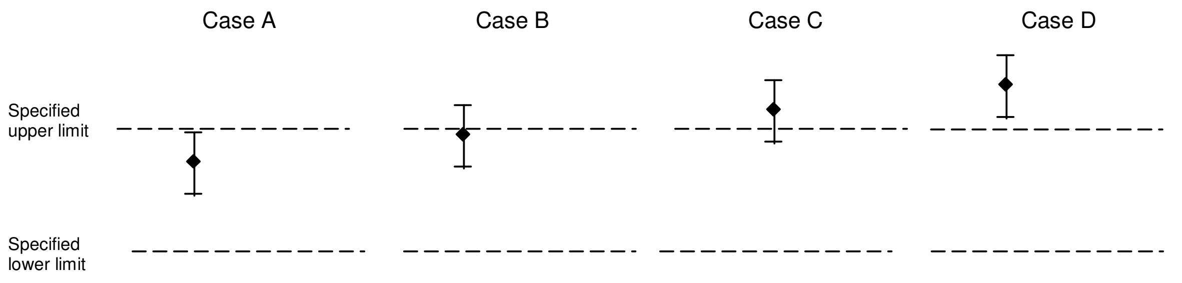
iii If a lower specification limit is breached even when the measured result is extended upwards by half of the expanded uncertainty interval, then non-compliance with the specification can be stated (Case H, Fig 2)

iv If a measured single value, without the possibility of testing more samples from the same unit of product, falls sufficiently close to a specification limit such that half of the expanded uncertainty interval overlaps the limit, it is not possible to confirm compliance or non-compliance at the stated level of confidence. The result and expanded uncertainty should be reported together with a statement indicating that neither compliance nor non-compliance was demonstrated.

A suitable statement to cover these situations (Cases B and C, Fig 1 and Cases F and G, Fig 2) would be for example:

The test result is above / below the specification limit by a margin less than the measurement uncertainty; it is therefore not possible to state compliance / non-compliance based on the 95% level of confidence. However, where a confidence level of less than 95% is acceptable, a compliance / non-compliance statement may be possible.

7.2 Certainly it is worthwhile giving some attention to the anticipated measurement uncertainty before performing tests, so that the number of results that fall in the region of uncertainty is minimised. The traditional rule of thumb employed is to use a measurement system capable of measuring with an uncertainty of 1/10 of the specification limit. This ratio is called the Test Uncertainty Ratio (TUR). Its principle use has been in providing a rationale for selection of test equipment without undertaking a complete analysis of the measurement system.



Case A: The measured result is within the limits, even when extended by the uncertainty interval.

The product therefore compiles with the specification.

Case B: The measured result is below the upper limit, but by a margin less than half of the uncertainty interval; it is therefore not possible to state compliance based on the 95% level of confidence.

However, the result indicates that compliance is more probable than non-compliance.

Case C: The measured result is above the upper limit, but by a margin less than half of the uncertainty interval; it is therefore not possible to state non-compliance based on the 95% level of confidence.

However, the result indicates that non-compliance is more probable than compliance.

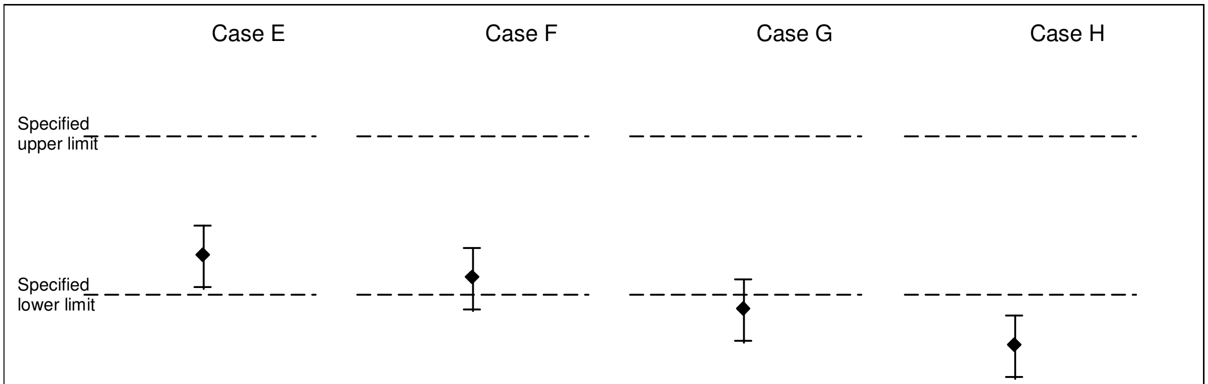
Case D: The measured result is beyond the upper limit, even when extended downwards by half of the uncertainly interval.

The product therefore does not comply with the specification.

◆ = measured result

⊥ = uncertainty interval

Fig 1 Assessing compliance when the result is close to an upper limit



Case E: The measured result is within the limits, even when extended by the uncertainty interval.

The product therefore compiles with the specification.

Case F: The measured result is above the lower limit, but by a margin less than half of the uncertainty interval; it is therefore not possible to state compliance based on the 95% level of confidence.

However, the result indicates that compliance is more probable than non-compliance.

Case G: The measured result is below the lower limit, but by a margin less than half of the uncertainty interval; it is therefore not possible to state non-compliance based on the 95% level of confidence.

However, the result indicates that non-compliance is more probable than compliance.

Case H: The measured result is beyond the lower limit, even when extended upwards by half of the uncertainly interval.

The product therefore does not comply with the specification.

◆ = measured result

⊥ = uncertainty interval

Fig 2 Assessing compliance where the result is close to a lower limit

8 REFERENCES

a) General Requirements for the Competence of Testing and Calibration Laboratories – ISO/IEC 17025: 2005

b) Guide to the Expression of Uncertainty in Measurement (GUM – 1995), Published by ISO

c) International Vocabulary of basic and general terms in Metrology (VIM – 1993), Published by ISO

d) Guidance for the use of repeatability, reproducibility and trueness estimates in measurement uncertainty estimation – ISO/TS 21748: 2004 (E)

e) Accuracy (trueness and precision) of Measurement Methods and Results Parts 1 to 6 – ISO 5725: 1994

f) Methods of Stating Test Results and Compliance with Specification – APLAC TC 004: 2004

g) Interpretation and Guidance on the Estimation of Uncertainty of Measurement in testing – APLAC TC 005 Issue No 2.

h) Assessment of Uncertainties of Measurement for Calibration & Testing Laboratories – NATA 2nd Edition 2002

i) EA Guidelines on the Expression of Uncertainty in Quantitative Testing – EA 4/16: Dec 2003

j) Standard Practice for Using Significant Digits in Test Data to Determine Conformance with Specifications - ASTM E29-2006

9. ACKNOWLEDGEMENT

The Singapore Accreditation Council would like to thank members of the Civil Engineering/ Mechanical Working Group and their respective organisations for their effort and contributions in establishing this Technical Guide.

The members of the Working Group are:

Chairman

Mr Edgar Sammy Lim Choon Teck	Keppel Offshore & Marine Technology Centre Pte Ltd

Member

Mr Kevin Quan	CPG Laboratories Pte Ltd
Ms Toh Seok Fern	Nanyang Polytechnic
Mr Raymond Koa	GBAD Services Pte Ltd

Secretary

Mr Lim Swee Peng	SAC-SINGLAS
Ms Karen Yeo	SAC-SINGLAS

APPENDIX A

GUM APPROACH

A1 GENERAL PRINCIPLES

A1.1 The **GUM** has adopted the approach of grouping uncertainty components into two categories based on their methods of evaluation, '**Type A**' and '**Type B**'.

- '**Type A**' evaluation is done by calculation from a series of repeated observations, using statistical methods.

- '**Type B**' evaluation is done by means other than that used for '**Type A**'. For example, by judgement based on data in calibration certificates, previous measurement data, experience with the behaviour of the instruments, manufacturers' specifications or all other relevant information.

A1.2 Components of uncertainty are evaluated by the appropriate method and each is expressed as a **standard deviation** and is referred to as a **standard uncertainty**.

A1.3 The standard uncertainty components are combined to produce an overall value of uncertainty, known as the **combined standard uncertainty**.

A1.4 An **expanded uncertainty** is usually required to meet the client's or regulatory requirement. It is intended to provide a greater interval about the result of a measurement than the standard uncertainty with, consequently, a higher probability that it encompasses the value of the measurand. It is obtained by multiplying the combined standard uncertainty by a **coverage factor**, *k*. The choice of factor is based on the coverage probability or **level of confidence** required.

A2 SOURCES OF UNCERTAINTY

A2.1 Some examples of the sources of uncertainty are given below:

(a) Sampling - the sample may not be fully representative

(b) Personal bias in reading analogue instruments

(c) Instrument resolution or discrimination threshold, or errors in graduation of a scale

(d) Uncertainty values assigned to measurement instruments, reference standards and reference materials

(e) Instrument drifting - changes in the characteristics or performance of a measuring instrument since the last calibration

(f) Variations in repeated observations made under apparently identical conditions – such random effects may be caused by, for example, short term fluctuations in local environment (temperature, humidity and air pressure), variability in the performance of the operator

A2.2 These sources as stated in A2.1 are not necessarily independent.

A2.3 Certain systematic effects may exist that cannot be taken into account though it contributed to the error. Such effects may be difficult to quantify and may be evident from examination of proficiency testing results, e.g. strain rate effect on tensile test result.

A2.4 The laboratory shall at least estimate all components of uncertainty and make a reasonable estimation.

A3 ESTIMATION OF UNCERTAINTY

A3.1 GUM Approach – Step by Step

A3.1.1 Uncertainty estimation is a straightforward process in principle. Some measurement processes are complex and hence the uncertainty calculations take on a degree of complexity. The overall principles remain the same.

A model of the measurement should be developed, either implicitly or explicitly depending on the complexity of the measurement. Next, all the uncertainty components are listed and their standard uncertainties calculated. Generally, it will be necessary to also calculate sensitivity coefficients. Sensitivity coefficients convert the components to the same units as the measurand, and also scale or weigh them so that they have the proper influence on the total uncertainty. Then the components are combined and an expanded uncertainty calculated for the measurand. These steps are dealt with in detail in the following sections and Worked Examples and summarized in Table 1.

A3.1.2 The total uncertainty of a measurement is a combination of a number of uncertainty components. A single instrument reading may even be influenced by several factors. Careful consideration of each measurement involved in the test is required so as to identify and list all the factors that contribute to the overall uncertainty. This is a very important step and requires a good understanding of the measuring equipment, the principles and practice of the test and the influence of environment.

A3.1.3 The next step is to quantify the uncertainty components by appropriate means. An initial approximate quantification may be valuable in enabling some components to be shown to be negligible and not worthy of more rigorous evaluation. In most cases, a practical guide would be that a component is negligible if it is not more than 5% of the total uncertainty. Some components may be quantified by calculation of the standard deviation from a set of repeated measurements (**Type A**) as detailed in the **GUM**. Quantification of others will require the exercise of judgement, using all relevant information on the possible variability of each factor (**Type B**).

For '**Type B**' estimations, the pool of information may include:

(a) previous measurement data;

(b) manufacturer's specifications;

(c) data provided in calibration certificates;

(d) uncertainty assigned to reference data taken from handbooks;

(e) experience with or general knowledge of the behaviour and properties of relevant materials and instruments; estimations made under this heading are quite common in many fields of testing.

A3.1.4 Whenever possible, corrections should be made for errors revealed by calibration or other sources; the convention is that an error is given a positive sign if the measured value is greater than the conventional true value. The correction for error involves subtracting the error from the measured value. On occasion, to simplify the measurement process it may be preferable to treat such an error, when it is small compared with other uncertainties, as if it were a systematic uncertainty equal to (±) the uncorrected error magnitude.

A3.1.5 Subsequent calculations will be made simpler if, wherever possible, all components are expressed in the same way, for example, either as a proportion **[percent (%)** or **parts per million (ppm)]** or in the **same units** as used for the reported result.

A3.1.6 The degree of rigor required in an estimation of measurement uncertainty depends primarily on the use of the test results and laboratory should ensure that the degree of rigor meets the client's requirements.

A3.1.7 Minor components that have been disregarded may need to be re-considered when a more rigorous estimation of measurement is required.

A3.1.8 Measurement uncertainty may need to be reviewed and revised when there are changes in the laboratory such as environmental conditions, equipment, calibration grading, personnel, etc.

A3.1.9 In some cases, the uncertainty associated in a measurement may be considered to be negligible. However, this consideration remains intuitive without a formal evaluation.

A3.2 Standard uncertainty

A3.2.1 The standard uncertainty is defined as the uncertainty of the result of a measurement expressed as a standard deviation. The potential for mistakes at a later stage of the evaluation may be minimized by expressing all component uncertainties as one standard deviation. This may require conversion of some uncertainty values, such as those obtained from calibration certificates and other sources that often will have been expressed to a different level of confidence, involving a multiple of the standard deviation.

A3.3 Combined standard uncertainty

A3.3.1 The component uncertainties have to be combined to produce an overall uncertainty using the procedure set out in the GUM. This may call for **partial differentiation** of the mathematical model of the measurands. The details of this mathematical derivation can be found in the GUM. In most cases where the components are independent, the root sum square (RSS) method can be used. If two or more systematic errors are correlated, that is they are not independent, then the RSS combination is not appropriate. The ISO GUM gives a detailed treatment for components that are correlated.

A3.4 Expanded uncertainty

A3.4.1 Expanded uncertainty defines an interval about the measurement result that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand. This is obtained by multiplying the combined standard uncertainty by an appropriate **coverage factor, k**. This coverage factor must reflect the **level of confidence** required, and, in strict terms, will be dictated by details of the probability distribution characterized by the measurement result and its combined standard uncertainty.

TABLE 1 THE SIX STEPS TO DETERMINING UNCERTAINTY OF MEASUREMENT

1. Make a model of the measurement system.
2. List all the sources of uncertainties.
3. Calculate the standard uncertainties for each component using type A analysis for those with repeated measurements and type B for others.
4. Calculate the sensitivity coefficients.
5. Calculate the combined uncertainty, and, if appropriate its effective degrees of freedom.
6. Calculate the expanded uncertainty. Use a calculated coverage factor. Round the measured value and the uncertainty to obtain the reported values.

WORKED EXAMPLES – BASED ON GUM APPROACH

The following generic worked examples are intended to show how the principles in this Technical Guide can be applied to the tests in the civil engineering and mechanical testing fields.

Example (A1) - Compressive Strength of Hardened Concrete Cubes

Example (A2) - Marshall Stability for Cored Premix Asphalt Sample

Example (A3) - Maximum Density of Gravelly Soils

Example (A4) - Tensile Strength of Metallic Materials

Example (A5) - Density of Hardened Concrete Cube Measurement Uncertainty Estimated using Numerical Differentiation

EXAMPLE (A1) - COMPRESSIVE STRENGTH OF HARDENED CONCRETE CUBE

A1.1. INTRODUCTION

This example serves to illustrate the estimation of measurement uncertainty according to Guide to the Expression of Uncertainty in Measurement (GUM: 1995) in the determination of compressive strength of hardened concrete cube tested to SS78: Part A16: 1987. The determination of density of the cubes is illustrated in a separate example.

Description of test: In this test a continuously increasing force is applied, using a loading machine, onto a hardened concrete cube until the specimen crushed. The strength, f_{cu} ,of the cube is calculated from the maximum applied force, F, divided by the cross-sectional area, A , of the load bearing face of dimensions, L_x and L_z , measured using a digital vernier caliper.

The working uses estimated uncertainties provided in calibration report and repeated observations of dimension measurements for each cubes.

.

A1.2. MODEL

Cube compressive strength, $f_{cu} = \dfrac{F}{A}$, Eq. 1

where F = maximum applied force on cube, and

A = cross sectional area,

$= L_{x,\,avg} \times L_{z,\,avg}$ Eq. 2

$L_{i,\,avg}$ = average of 2 pairs of orthogonal dimensions perpendicular to direction of loading

A1.3. RESULTS OF MEASUREMENT

Max. Force F (kN)	Orthogonal dimensions (mm)							Area, A (mm^2)	Strength, (N/mm^2) f_{cu}
	Dir	pair #1		pair #2		Average $L_{i,\,avg}$	Std Dev		
1040	L_x	150.67	149.87	149.60	149.39	149.882	0.561	22666.7	45.882
	L_z	151.38	151.38	151.08	151.08	151.230	0.173		

Note: For this example, all text in italics serve as explanatory notes only.

A1.4. SOURCES OF UNCERTAINTY

Factors	Source of Uncertainty	Remarks
Loading machine	Deviation of reading from nominal	Estimated by regular calibration within equip. specs.
	Loading rate	Not considered – maintained within recommendation
	Stiffness of machine	Not considered – within recommendation
	Alignment of loading patens	Not considered – within recommendation
	Operating temperature	Not considered – maintained within recommendation
Vernier Caliper (digital)	Accuracy of equipment	Estimated by regular calibration within equip. specs.
	Resolution of observed output	Estimated from equipment stability and resolution
	Repeatability of reading	Estimated by regular in house operator verification
	Operating temperature	Not considered – maintained within recommendation
Test specimen	Inhomogeneity among specimens	Not considered – require repeated testing
	Dimensional variation of cube faces	Estimated by repeated measurements
	Perpendicularity of cube face	Not considered – within recommendation
	Roughness of loading surface	Not considered – within recommendation
	Moisture condition of cubes	Not considered – maintained within recommendation

A1.5. ESTIMATION OF STANDARD UNCERTAINTY

a) Force measurement:

a. i) Deviation of reading from nominal,

From report of calibration done on site, the estimated uncertainty of the machine indication, taking into consideration the indicator resolution, zero error, repeatability and the applied load uncertainty, is calculated to be ± 2.6kN defined at an approximated 95% confidence level with coverage factor k =2.13.

Note : Ensure that equipment is performing within equipment specification at time of use

Note : If correction is required, errors ought to be corrected by interpolating between calibration points. Conservatively, errors may also be treated as uncertainty within a range derived from historical data, or experience. Uncertainties listed in equipment specification may also be use to cover all general cases, provided equipment performs within specification during time of use.

Standard uncertainty of applied force, $u_F \quad = \quad \dfrac{2.6}{2.13} \quad = \quad 1.22$kN

Type B evaluation *based on confidence level of 95%*

Degree of freedom, $\nu_F \quad = \quad 15$ *at k = 2.13 and consulting T-distribution table*

.

b) Dimension measurement

b. i) Accuracy of equipment,

From report of calibration, the estimated uncertainty of the measurement to be associated with internal measurement of the vernier caliper is ±0.01mm defined at an approximated 95% confidence level with coverage factor k =2.

Note : Usually calibration is <u>not</u> performed to site condition, ensure that site condition does not introduce significant deviation from the calibration environment, else such errors has to be accounted.

Standard uncertainty from calibration, $u_{Lc} = \frac{0.01}{2} = 0.005\text{mm}$

Type B evaluation based on confidence level of 95% at k=2
Degree of freedom, $\nu_{Lc} = 60$

b. ii) Resolution of observed output,

Resolution of the equipment is 0.01mm, assuming rectangular distribution.

Note : *This accounts for the rounding off of data. In digital output, this is introduced by the internal circuitry or software programme.*

Standard uncertainty of resolution, $u_{Lr} = \frac{0.01/2}{\sqrt{3}} = 0.003\text{mm}$

Type B evaluation based on assumed probability distribution

Degree of freedom, $\nu_{Lr} = \infty$

b. iii) Repeatability of reading

From regular in-house operator verification using the same vernier caliper verified against gauge block of representative size, the standard deviation of 30 repeated readings is ± 0.02mm.

Standard uncertainty of reading, $u_{Lo} = \frac{0.02}{\sqrt{30}} = 0.004\text{mm}$

Type A evaluation
Degree of freedom, $\nu_{Lo} = 30\text{-}1 = 29$

c) Test Specimen

c.i) Dimensional variation of cube faces

No. of reading for each dimension, = 4

Standard deviation of reading for L_x = 0.561mm

Standard uncertainty of $L_{x,avg}$, $u_{Lx} = \frac{0.561}{\sqrt{4}} = 0.280\text{mm}$

Standard deviation of reading for L_z = 0.173mm

Standard uncertainty of $L_{z,\,avg}$, $u_{Lz} = \frac{0.173}{\sqrt{4}} = 0.086\text{mm}$

Type A evaluation,
Degree of freedom, $\nu_{L,r} = n-1 = 4-1 = 3$

Therefore the combined standard uncertainty of dimension measurement of $L_{x,\,avg}$

$$u_{Lx,avg} = \sqrt{u_{Lc}{}^2 + u_{Lr}{}^2 + u_{Lo}{}^2 + u_{Lx}{}^2} = \sqrt{0.005^2 + 0.003^2 + 0.004^2 + 0.280^2}$$

$$= 0.280\text{mm}$$

And the combined standard uncertainty of dimension measurement of $L_{x,\,avg}$

$$u_{Lz,avg} = \sqrt{u_{Lc}{}^2 + u_{Lr}{}^2 + u_{Lo}{}^2 + u_{Lz}{}^2} = \sqrt{0.005^2 + 0.003^2 + 0.004^2 + 0.086^2}$$

$$= 0.086\text{mm}$$

A1.6. COMBINED STANDARD UNCERTAINTY

From Equation (2):

Sensitivity coefficient of length $L_{x,\,avg}$, $c_{Lx,avg} = \frac{\partial A}{\partial L_{x,\,avg}} = L_{z,\,avg} = 151.230\text{mm}$

Sensitivity coefficient of length $L_{z,\,avg}$, $c_{Lz,avg} = \frac{\partial A}{\partial L_{z,\,avg}} = L_{x,\,avg} = 149.882\text{mm}$

The combined standard uncertainty of area, u_A

$$u_A = \sqrt{c_{Lx,avg}{}^2 \times u_{Lx,avg}{}^2 + c_{Lz,avg}{}^2 \times u_{Lz,avg}{}^2}$$

$$= \sqrt{151.230^2 \times 0.280^2 + 149.882^2 \times 0.086^2}$$

$$= 44.3\text{mm}^2$$

From Equation (1):

Sensitivity coefficient of force, $c_F = \frac{\partial f_{cu}}{\partial F} = \frac{1}{A}$

$$= \frac{1}{22666.7} = 4.41175 \times 10^{-5}/\text{mm}^2$$

Sensitivity coefficient of area, $c_A = \dfrac{\partial f_{cu}}{\partial A} = -\dfrac{F}{A^2}$

$$= -\frac{1040\times10^3}{22666.7^2} = -2.0242\times10^{-3}\text{N/mm}^4$$

Therefore, the combined standard uncertainty of strength, u_c

$$u_c = \sqrt{c_F{}^2 \times u_F{}^2 + c_A{}^2 \times u_A{}^2}$$

$$= \sqrt{(4.41175\times10^{-5})^2\times(1.22\times10^3)^2+(-2.0242\times10^{-3})^2\times(44.3)^2}$$

$$= 0.104\text{N/mm}^2$$

A1.7. ESTIMATE OF EXPANDED UNCERTAINTY

Coverage factor,

Effective degree of freedom of dimension measurement $L_{x,\,avg}$,

$$\nu_{x,\,avg} = \frac{u_{Lx,avg}{}^4}{\sum_{i=1}^{N}\frac{c_{L,i}{}^4 u_{L,i}{}^4}{\nu_{L,i}}} = \frac{u_{Lx,avg}{}^4}{\left(\frac{c_{Lc}{}^4 u_{Lc}{}^4}{\nu_{Lc}}+\frac{c_{Lr}{}^4 u_{Lr}{}^4}{\nu_{Lr}}+\frac{c_{Lo}{}^4 u_{Lo}{}^4}{\nu_{Lo}}+\frac{c_{Lx}{}^4 u_{Lx}{}^4}{\nu_{Lx}}\right)}$$

$$= \frac{0.280^4}{\left(\frac{1^4\times0.005^4}{60}+\frac{1^4\times0.003^4}{\propto}+\frac{1^4\times0.004^4}{29}+\frac{1^4\times0.280^4}{3}\right)}$$

$$= 3.00$$

Effective degree of freedom of dimension measurement $L_{z,\,avg}$,

$$\nu_{z,\,avg} = \frac{u_{Lz,avg}{}^4}{\sum_{i=1}^{N}\frac{c_{L,i}{}^4 u_{L,i}{}^4}{\nu_{L,i}}} = \frac{u_{Lz,avg}{}^4}{\left(\frac{c_{Lc}{}^4 u_{Lc}{}^4}{\nu_{Lc}}+\frac{c_{Lr}{}^4 u_{Lr}{}^4}{\nu_{Lr}}+\frac{c_{Lo}{}^4 u_{Lo}{}^4}{\nu_{Lo}}+\frac{c_{Lz}{}^4 u_{Lz}{}^4}{\nu_{Lz}}\right)}$$

$$= \frac{0.086^4}{\left(\frac{1^4\times0.005^4}{60}+\frac{1^4\times0.003^4}{\propto}+\frac{1^4\times0.004^4}{29}+\frac{1^4\times0.086^4}{3}\right)}$$

$$= 3.00$$

Effective degree of freedom of area, ν_A

$$\nu_A = \frac{u_A{}^4}{\sum_{i=1}^{N}\frac{c_{L,i}{}^4 u_{L,i}{}^4}{\nu_{L,i}}} = \frac{u_A{}^4}{\left(\frac{c_{Lx,avg}{}^4 u_{Lx,avg}{}^4}{\nu_{Lx,avg}}+\frac{c_{Lz,avg}{}^4 u_{Lz,avg}{}^4}{\nu_{Lz,avg}}\right)}$$

$$\nu_A = \frac{44.3^4}{\left(\frac{151.230^4 \times 0.280^4}{3.00} + \frac{149.882^4 \times 0.086^4}{3.00}\right)}$$

$$= 3.56$$

Effective degree of freedom of strength, ν_{eff}

$$\nu_{eff} = \frac{u_c^{\ 4}}{\sum_{i=1}^{N} \frac{c_i^{\ 4} u_i^{\ 4}}{\nu_i}} = \frac{u_c^{\ 4}}{\left(\frac{c_F^{\ 4} u_F^{\ 4}}{\nu_F} + \frac{c_A^{\ 4} u_A^{\ 4}}{\nu_A}\right)}$$

$$= \frac{0.104^4}{\left(\frac{(4.41175 \times 10^{-5})^4 \times (1.22 \times 10^3)^4}{15} + \frac{(-2.0242 \times 10^{-3})^4 \times 44.3^4}{3.56}\right)}$$

$$= 6.24$$

k = 2.43 at 95% level of confidence, *from t-distribution table*

Expanded uncertainty, $U = k\,u_c$
$= 2.43 \times 0.104 = 0.252\text{N/mm}^2$

Therefore the uncertainty in the cube compressive strength is 0.252N/mm^2

A1.8. REPORTING OF RESULTS

Cube compressive strength, f_{cu} = 45.88 ± 0.25N/mm^2 at level of confidence of 95% (k=2.43)

Therefore, the cube compressive strength is: 46.0N/mm^2 tested in accordance to SS78: Part A16:1987.

The test method requires rounding to nearest 0.5N/mm^2, without quoting the measurement uncertainty.

EXAMPLE (A2) - MARSHALL STABILITY FOR CORED PREMIX ASPHALT SAMPLE

A2.1. INTRODUCTION

This example serves to illustrate the estimation of measurement uncertainty according to ISO Guide to the Expression of Uncertainty in Measurement (GUM: 1995) in the determination of Marshall Stability for Cored Premix Asphalt Sample.

The Marshall Stability Test is in accordance with: ASTM D1559 –1989 "Standard Test Method for Resistance to Plastic Flow of Bituminous Mixtures Using Marshall Apparatus"

A2.2. MODEL

The Marshall Stability is recorded as the maximum load reached when the sample is loaded by means of constant rate of movement of the testing-machine head multiplied by correlation ratio provided by the standard (the normalised Marshall Stability value is based on a 101.6mm diameter and 63.5mm thick sample);

Marshall Stability (corrected) = F x C.R. …..(1)

where F, is the force reading from the Marshall Stability Tester at failure load of the Marshall Sample.

C.R., is the correlation ratio obtained from Table 1 "Stability Correlation Ratios" of ASTM D1559. The correlation ratio is determined by measuring the volume of the cored sample.

A2.3. RESULT OF MEASUREMENT

A core sample is submitted to the laboratory and to be tested as received.

Sample measurements:

Dry mass of the sample (Mass in air), M1 = 1196.0 g
(Applying estimated correction => 1196.0)

Mass of sample when immersed in water (Mass in water), M2 = 678.1 g
(Applying estimated correction => 678.1)

Volume of sample, V (M1 – M2) = 517.9 cm^3

From Table 1 "Stability Correlation Ratios" of ASTM D1559, Correlation Ratio, C.R. = 1.00

Measured maximum load, F = 15180 N
= 1.5180×10^4 N

Marshall Stability (corrected) = F x C.R.
= 1.52×10^4 N

A2.4. SOURCES OF UNCERTAINTY

There are several influencing parameters that affect the uncertainty of Marshall Stability measurement, and are grouped as follows in Table 1:

Table 1: Sources of Uncertainty

S/No	Influencing Factor	Source of Uncertainty	Remarks
1.	Test Sample	Size of Core Extracted	Sample extraction (The standard does not specify the sample size tolerance but provides a correlation ratio. Table 1 of ASTM D1559 based on sample volume)
		Sample Temperature at test	Sample Preparation – standard procedures to be adhered to strictly and kept within allowable tolerances as specified in standard
2.	Marshall Compression Machine	Deviation from nominal	Calibration/Machine Spec
		Placement of sample in machine	Unable to assess and also minor
		Rate of loading	Unable to assess based on available data. However, assumed verified to be within tolerance through calibration.
3.	Weighing Balance (For determining volume of sample)	Deviation from nominal	Calibration

A2.5. ESTIMATION OF STANDARD UNCERTAINTY

a) Standard Uncertainty of Force measurement:

Uncertainty in calibration of Marshall Tester, (instrument uncertainty)
Given that the Marshall Compression machine is a Class 1 machine;

=> Standard uncertainty, $u_{F,t} = \dfrac{1\% \times 15180}{\sqrt{3}} = 88N$

Assuming Rectangular Distribution;

=> Degree of Freedom, $\nu_{F,t}$ = infinity

b) Standard Uncertainty of Correlation Ratio (Mass measurement ie mass in air less mass in water, is used to calculate the volume of the sample, which is the basis for determination of the correlation ratio):

Uncertainty in calibration of weighing balance, (instrument uncertainty)
Given that the calibration report for the electronic weighing balance used states the measurement uncertainty of the balance as $\pm$ 0.1 g at a confidence level of not less than 95%;

=> Standard uncertainty, $u_{C.R.,t} = \dfrac{0.1}{2} = 0.05$ g

Assuming Normal Distribution with coverage factor k=1.96;

=> Degree of Freedom, $\nu_{C.R.,t}$ = infinity

Note – This information should be derived from the calibration report.

A2.6. ESTIMATION OF COMBINED STANDARD UNCERTAINTY

From Equation (1);

Marshall Stability (corrected) = F x C.R.

The combined standard uncertainty of the Marshall Stability, $u_{Marshall}$,

$$u_{Marshall} = \sqrt{c_{F,t}{}^2 u_{F,t}{}^2 + c_{C.R.,t}{}^2 u_{C.R.,t}{}^2}$$

Where $c_{F,t}$ = Sensitivity coefficient of Force

$$= \frac{\partial MS}{\partial F}$$

= C.R. = 1.00

$c_{C.R.,t}$ = Sensitivity coefficient of Correlation Ratio

$$= \frac{\partial MS}{\partial C.R.}$$

= F = 1.5180x10^4 N

Therefore, $U_{Marshall} = \sqrt{1.00^2 x 88^2 + (1.5180 x 10^4)^2 x 0.05^2}$

= 7.6x10^2 N

A2.7. ESTIMATION OF EXPANDED UNCERTAINTY

Effective degree of freedom, ν_{eff}

$$\nu_{eff} = \frac{u_{Marshall}{}^4}{\sum_{i=1}^{N} \frac{(c_i \; u_i)^4}{\nu_i}} = \text{infinity}$$

=> Coverage factor, k = 1.96 at 95% level of confidence (from Student-t Distribution)

Expanded uncertainty, $U = k\, u_{Marshall}$
$= 1.96 \times 7.6 \times 10^2$ N
$= 1.5 \times 10^3$ N

A2.8. REPORTING OF RESULTS

Therefore, the Marshall Stability Value = 15180 ± 1500 N at 95% confidence level with a coverage factor of k=1.96.

(Dropping the intermediate significant digit, the Marshall Stability Value = 15180 ± 2000 N at 95% confidence level with a coverage factor of k=1.96)

The reported uncertainty is an expanded uncertainty with a coverage factor of k=1.96, which provides a level of confidence of 95%, but excluding the effects of rate of loading and sample preparation.

Unit sample was delivered by the client to the laboratory, as such, sampling uncertainty is not included in the expanded uncertainty.

ANNEX

UNCERTAINTY BUDGET TABLE

Source of Uncertainty	Symbol	Type	Uncertainty Value	Probability Distribution	Coverage Factor	Standard Uncertain-ty	Sensitivity Coeff.	C_i x $u_{F,i}$	Degree of Freedom
Marshall Machine Force Reading	$u_{F,t}$	B	1% of force	Rectangular	-	88 N	1.00	88	Infinity
Correlation Ratio as determined by sample volume (based on mass measure-ment)	$U_{C.R.,t}$	B	$\pm$ 0.1 g	Normal	-	0.05	1.5180 x10^4	7.6 x10^2	Infinity

EXAMPLE (A3) - MAXIMUM DENSITY OF GRAVELLY SOILS

A3.1. INTRODUCTION

This example serves to illustrate the estimation of measurement uncertainty according to ISO Guide to the Expression of Uncertainty in Measurement (GUM: 1995) in the determination of Maximum Density of Gravelly Soils.

The Maximum Density of Gravelly Soils is determined in accordance with BS1377: Part4 : 1990 Section 4.3.

A3.2. MODEL

The test covers the determination of the maximum density to which a gravel or sandy gravel can be compacted. The soil is compacted into a 152mm diameter CBR mould using an electric vibrating hammer. The volume of the mould is determined by measuring the dimensions of the mould.

The compacted soil in the mould is extracted and oven dried before weighing to determine its mass. The maximum density is determined as follows:

Maximum Density, ρ max $= \dfrac{m}{V}$ .....(1)

where m, is mass of the soil compacted into the mould, weighed after oven drying (recorded to 5g).

V, is the calculated volume of the mould based on measured dimensions.

$$V = \frac{\pi D^2 x H}{4}$$

Where D, is the measured diameter of the mould

H, is the measured height of the mould

A3.3. RESULT OF MEASUREMENT

Soil material that is appropriate for the test is submitted to the lab and tested as received.

Measurements:

Mass of compacted soil after oven drying, m = 4275 g

Dimension of mould:

Repetition	Diameter, D (mm)	Height, H (mm)
1	151.94	127.07
2	151.73	126.87
3	152.22	126.86
4	151.74	127.01
Ave	151.908	126.952

Volume of mould, $V = \frac{\pi 0.151908^2 x 0.126952}{4}$ m^3

$= 0.00230086\ m^3$

Maximum Density, ρ max $= \frac{4275}{0.00230086}$

$= 1.858\ Mg/\ m^3$

A3.4. SOURCES OF UNCERTAINTY

There are several influencing parameters that affect the uncertainty of the maximum density determination for gravelly soils, and are grouped as follows in Table 1:

Table 1: Sources of Uncertainty

S/No	Influencing Factor	Source of Uncertainty	Remarks
1.	Test Sample	Sampling procedure	Sampling procedure from the source must be appropriate to obtain representative sample. As sampling was not carried out by the lab, the sampling uncertainty is not included.
2.	Test Procedure	Deviations from specified procedures to be followed	It is deemed that the specified procedures have been followed closely and without deviation. Unable to quantify the uncertainty based on the data available.
3.	Weighing Balance	Deviation from nominal	Calibration
4.	Digital vernier caliper	Deviation from nominal	Calibration

A3.5. ESTIMATION OF STANDARD UNCERTAINTY

a) Standard Uncertainty of Mass measurement:

Uncertainty in calibration of weighing balance, (instrument uncertainty). Given that the calibration report for the electronic weighing balance used states the expanded uncertainty to be associated with the balance as $\pm$ 0.1 g at a confidence level of approximately 95% with a coverage factor of k=2.0;

=> Standard uncertainty associated with balance, $u_{.M.} = \frac{0.1}{2.0} = 0.05$ g

=> Degree of Freedom, $\nu_{.M.}$ = infinity

Note : This information should be derived from the calibration report

b) Standard Uncertainty of dimensional measurements used to compute volume of mould:

Uncertainty in calibration of digital caliper, (instrument uncertainty). Given that the calibration report for the digital caliper used states the measurement uncertainty of the caliper as $\pm$ 0.01 mm at a confidence level of approximately 95% with coverage factor k=2.0

=> Standard uncertainty associated with caliper, $u_{.C.,} = \dfrac{0.01}{2.0}$ = 0.005 mm

=> Degree of Freedom, $\nu_{C.,}$ = infinity

Note : This information should be derived from the calibration report

c) Dimensional variation of mould:

Number of readings for each dimension = 4

Standard Deviation of reading for diameter, D = 0.230 mm

Standard Uncertainty of diameter, D $u_D = \dfrac{0.230}{\sqrt{4}}$ = 0.115 mm

Standard Deviation of reading for height, H = 0.104 mm

Standard Uncertainty of height, H $u_H = \dfrac{0.104}{\sqrt{4}}$ = 5.20 x 10^{-2} mm

Type A evaluation,
=> Degree of Freedom, $\nu_{D, H.,}$ = n – 1 = 4 – 1 = 3

A3.6. ESTIMATION OF COMBINED UNCERTAINTY

Combined standard uncertainty of dimensional measurement for diameter, D

$$U_D = \sqrt{u_C{}^2 + u_D{}^2} = \sqrt{0.005^2 + 0.115^2}$$

$$= 0.12 \text{ mm}$$

Combined standard uncertainty of dimensional measurement for height, H

$$U_H = \sqrt{u_C{}^2 + u_H{}^2} = \sqrt{0.005^2 + 0.052^2}$$

$$= 0.052 \text{ mm}$$

From Equation (1);

Maximum Density, ρ max = $\frac{\mathrm{m}}{V}$ …..(1)

$$= \frac{4m}{\pi D^2 x H}$$

The combined standard uncertainty of the maximum density, ρ max,

$U\rho_{max}$ = $\sqrt{c_m{}^2 u_m{}^2 + c_D{}^2 u_D{}^2 + c_H{}^2 u_H{}^2}$

Where c_m = Sensitivity coefficient of mass

$$= \frac{\partial \rho}{\partial m}$$

$$= \frac{4}{\pi D^2 x H}$$

$$= \frac{4}{\pi 0.151908^2 x 0.126952}$$

= 434.620

c_D = Sensitivity coefficient of mould diameter

$$= \frac{\partial \rho}{\partial D}$$

$$= \frac{-8m}{\pi D^3 x H}$$

$$= \frac{-8x4275}{\pi 0.151908^3 x 0.126952}$$

= -2.4462 x 10^7

c_H = Sensitivity coefficient of mould height

$$= \frac{\partial \rho}{\partial H}$$

$$= \frac{-4m}{\pi D^2 x H^2}$$

$$= \frac{-4x4275}{\pi 0.151908^2 x 0.126952^2}$$

$$= -1.4635 \times 10^7$$

Therefore, $U\rho_{max} = \sqrt{(434.620)^2 x 0.05^2 + (-2.4462 x 10^7)^2 x 0.00012^2 + (-1.4635 x 10^7)^2 x 0.00005^2}$

= 2914 g/m^3

= 0.0029 Mg/m^3

A3.7. ESTIMATION OF EXPANDED UNCERTAINTY

Effective degree of freedom, ν_{eff}

$$\nu_{eff} = \frac{u\rho_{max}{}^4}{\sum_{i=1}^{N} \frac{(c_i\ u_i)^4}{\nu_i}}$$

$$= \frac{2900^4}{0 + \frac{[(-2.4462 \times 10^7)0.00012]^4}{3} + \frac{[(-1.4635 \times 10^7)0.00005]^4}{3}}$$

= 2.8

=> Coverage factor, k = 4.30 at 95% level of confidence (from Student's t-Distribution)

Expanded uncertainty, U = k $U\rho_{max}$

= 4.30 x 0.0029 Mg/m^3

= 0.012 Mg/m^3

A3.8. REPORTING OF RESULTS

Therefore, the maximum density, ρ max = 1.86 ± 0.01 Mg/m^3 at 95% confidence level with a coverage factor of k=4.30.

The reported uncertainty is an expanded uncertainty with a coverage factor of k=4.30, which provides a level of confidence of 95%, but excluding the uncertainty from sampling and deviations from test procedure (assumed specified procedures carried out accurately).

Unit sample was delivered by the client to the laboratory, as such, sampling uncertainty is not included in the expanded uncertainty.

EXAMPLE (A4) - TENSILE STRENGTH OF METALLIC MATERIAL

A4.1. INTRODUCTION

This example serves to illustrate the estimation of measurement uncertainty according to Guide to the Expression of Uncertainty in Measurement (GUM: 1995) in the determination of tensile test for metallic materials (a steel rectangular bar in this example).

The tensile test method is based on:
ASTM A370 – 02 "Standard Test Method and Definitions for Mechanical Testing of Steel Products"

A4.2. MODEL

The tensile strength is a function of force applied (max) and the cross-sectional area. The formula being represented as follows:

$$\sigma_{TS} = \frac{F}{A}$$

where σ_{TS} = Tensile Strength
F = Max Load
A = Cross-sectional Area

For rectangular specimen: $A = w \times t$ (w = width; t = thickness)

A4.3. RESULTS OF MEASUREMENT

A steel plate sample was submitted by the client to the laboratory. The sample was prepared and a tensile test was conducted as per test standard. The test results obtained were presented in Table 1.

Table 1: Tensile Test Results on Steel Plate

Measurement	Results
Width, w (mm)	12.55 12.58 12.56
	Ave: 12.563 Std Dev. 0.015
Thickness, t (mm)	5.11 5.13 5.15
	Ave: 5.130 Std Dev. 0.020
Cross-sectional Area, A (mm^2)	64.45
Maximum Load, F (N)	48598
Tensile Strength, σ (Nmm^{-2})	**754.0**

The specification provided for the submitted sample was minimum tensile strength of **750 Nmm^{-2}**.

A4.4. SOURCES OF UNCERTAINTY

There are several influencing parameters that affect the uncertainty of tensile strength measurement, and are grouped as follows in Table 2:

Table 2 : Sources of Uncertainty

S/No.	Influencing Factor	Source of Uncertainty	Remarks
1	Test Piece	Machining tolerance	Sample Preparation
		Sample homogeneity	Not considered
2.	Universal Tensile Machine	Deviation from nominal	Calibration
		Method of clamping	Not considered; Deemed to be minor as using standard dumbell sample
		Rate of loading	Not considered; Steel material is not so sensitive as long as within range specified in test method.
		Stiffness of machine	Not considered; Difficult to quantify
3.	Vernier Caliper / Micrometer	Deviation from nominal	Calibration
4.	Environment	Temperature deviation	Not considered; Steel material is not so sensitive for small range of temperature changes at ambient temperature.
5.	Operator	Handling, reading, evaluation errors	Operator error - Not considered

A4.5. ESTIMATION OF STANDARD UNCERTAINTY

4.5.1 Standard Uncertainty of Force

<u>Calibration of Testing Machine (Force)</u>

From the calibration report of the universal tensile machine's load cell, it is calibrated to Grade 1 (±1% error). This is taken as the uncertainty (since no correction is applied to the result), assuming a Rectangular distribution.

Hence the **standard uncertainty of force (max)** is

$$u_F = \frac{48598 \times 1\%}{\sqrt{3}}$$

$$= 280\,N$$

The degree of freedom, $v_F = \infty$

4.5.2 Standard Uncertainty of Width

<u>Calibration of Point Micrometer</u>

From the calibration report of the micrometer, the error of the instrument was 0.002mm and met the equipment specifications of accuracy of 0.003mm. [No correction applied to the instrument reading.]

Hence, assuming rectangular distribution, the standard uncertainty of micrometer is

$$u_{micrometer,w} = \frac{0.003}{\sqrt{3}}$$
$$= 0.0017mm$$

The degree of freedom, $v_{micrometer,w} = \infty$

<u>Operator Observation</u>

This is taken as the resolution of the micrometer, and assuming a rectangular distribution.

Resolution of micrometer = 0.01mm

Assuming a rectangular distribution, the standard uncertainty associated with operator is given as:

$$u_{operator,w} = \frac{0.01}{2} \times \frac{1}{\sqrt{3}}$$
$$= 0.0029mm$$

The degree of freedom, $v_{operator,w} = \infty$

<u>Sample Preparation</u>

The Type A standard uncertainty of the width arising from repeated measurements on the sample is determined to be:

$$u_{sample,w} = \frac{S(w)}{\sqrt{n_w}}$$
$$= \frac{0.015}{\sqrt{3}}$$
$$= 0.0087mm$$

where S(***w***): Standard deviation of width measurements made on sample
n

n_w: No. of width measurements made on sample

The degree of freedom, $v_{sample} = n - 1 = 3 - 1 = 2$

Standard Uncertainty of Width (Combined)

The **standard uncertainty of width** is therefore determined as:

$$
\begin{aligned}
u_w &= \sqrt{u_{micrometer,w}^{\ \ 2} + u_{operator,w}^{\ \ 2} + u_{sample,w}^{\ \ 2}} \\
&= \sqrt{0.0017^2 + 0.0029^2 + 0.0087^2} \\
&= 0.0093mm
\end{aligned}
$$

The effective degree of freedom is

$$
\begin{aligned}
v_w &= \frac{u_w^{\ 4}}{\dfrac{u_{micrometer,w}^{\ \ 4}}{v_{micrometer,w}} + \dfrac{u_{operator,w}^{\ \ 4}}{v_{operator,w}} + \dfrac{u_{sample,w}^{\ \ 4}}{v_{sample,w}}} \\
&= \frac{u_w^{\ 4}}{u_{sample,w}^{\ \ 4}} \times v_{sample,w} \\
&= \frac{0.0093^4}{0.0087^4} \times 2 \\
&= 2.6
\end{aligned}
$$

4.5.3 Standard Uncertainty of Thickness

Calibration of Point Micrometer

From the calibration report of the micrometer, the error of the instrument was 0.002mm and met the equipment specifications of accuracy of 0.003mm. [No correction applied to the instrument reading.]

Hence, assuming rectangular distribution, the standard uncertainty of micrometer is

$$
\begin{aligned}
u_{micrometer,t} &= \frac{0.003}{\sqrt{3}} \\
&= 0.0017mm
\end{aligned}
$$

The degree of freedom, $v_{micrometer,t} = \infty$

Operator Error

This is taken as the resolution of the micrometer, and assuming a rectangular distribution.

Resolution of micrometer = 0.01mm

Assuming a rectangular distribution, the standard uncertainty associated with operator is given as:

$$
\begin{aligned}
u_{operator,t} &= \frac{0.01}{2} \times \frac{1}{\sqrt{3}} \\
&= 0.0029mm
\end{aligned}
$$

The degree of freedom, $v_{operator,t} = \infty$

Sample Preparation

The Type A standard uncertainty of the thickness arising from repeated measurements on the sample is determined to be:

$$\begin{aligned} u_{sample,t} &= \frac{S(t)}{\sqrt{n_t}} \\ &= \frac{0.020}{\sqrt{3}} \\ &= 0.012mm \end{aligned}$$

where S(***t***): Standard deviation of thickness measurements made on sample
$\boldsymbol{n_t}$: No. of thickness measurements made on sample

The degree of freedom, $v_{sample,t} = n - 1 = 3 - 1 = 2$

Standard Uncertainty of Thickness (Combined)

The **standard uncertainty of thickness** is therefore determined as:

$$\begin{aligned} u_t &= \sqrt{u_{micrometer,t}^{\;2} + u_{operator,t}^{\;2} + u_{sample,t}^{\;2}} \\ &= \sqrt{0.0017^2 + 0.0029^2 + 0.012^2} \\ &= 0.012mm \end{aligned}$$

The effective degree of freedom is

$$\begin{aligned} v_t &= \frac{u_t^{\;4}}{\frac{u_{micrometer,t}^{\;4}}{v_{micrometer,t}} + \frac{u_{operator,t}^{\;4}}{v_{operator,t}} + \frac{u_{sample,t}^{\;4}}{v_{sample,t}}} \\ &= \frac{u_t^{\;4}}{u_{sample,t}^{\;4}} \times v_{sample,t} \\ &= \frac{0.012^4}{0.012^4} \times 2 \\ &= 2.0 \end{aligned}$$

A4.6. ESTIMATION OF COMBINED STANDARD UNCERTAINTY

From

$$\sigma_{TS} = \frac{F}{A} = \frac{F}{w \times t}$$

Then the **combined standard uncertainty of tensile strength** is given by:

$$u_c = \sqrt{c_F{}^2 u_F{}^2 + c_w{}^2 u_w{}^2 + c_t{}^2 u_t{}^2}$$

where

$$\begin{aligned} c_F &= \text{Sensitivity Coef of Force} \\ &= \frac{\partial \sigma}{\partial F} = \frac{1}{w \times t} = \frac{1}{12.563 \times 5.130} \\ &= 0.01552 \end{aligned}$$

$$\begin{aligned} c_w &= \text{Sensitivity Coef of Width} \\ &= \frac{\partial \sigma}{\partial \mathrm{w}} = \frac{-F}{w^2 \times t} = \frac{-48598}{12.563^2 \times 5.130} \\ &= -60.02 \end{aligned}$$

$$\begin{aligned} c_t &= \text{Sensitivity Coef of Thickness} \\ &= \frac{\partial \sigma}{\partial \mathrm{t}} = \frac{-F}{w \times t^2} = \frac{-48598}{12.563 \times 5.130^2} \\ &= -147.0 \end{aligned}$$

Therefore,

$$\begin{aligned} u_c &= \sqrt{0.01552^2 \times 280^2 + (-60.02)^2 \times 0.0093^2 + (-147.0)^2 \times 0.012^2} \\ &= 4.7\,Nmm^{-2} \end{aligned}$$

The effective degree of freedom is given by:

$$\begin{aligned} v_{eff} &= \frac{u_c{}^4}{\frac{(c_F u_F)^4}{v_F} + \frac{(c_w u_w)^4}{v_w} + \frac{(c_t u_t)^4}{v_t}} \\ &= \frac{4.7^4}{\frac{(0.01552 \times 280)^4}{\infty} + \frac{(-60.02 \times 0.0093)^4}{2.6} + \frac{(-147.0 \times 0.012)^4}{2.0}} \\ &= 100 \end{aligned}$$

A4.7. ESTIMATION OF EXPANDED UNCERTAINTY

With v_{eff} = 100 and assuming a t-distribution, the coverage factor is k = 1.96 at 95% confidence level.

Hence the **Expanded Uncertainty** is determined as:

$$\begin{aligned} U_{\exp} &= k \bullet u_C \\ &= 1.96 \times 4.7 \\ &= \pm 9.2\,Nmm^{-2} \end{aligned}$$

at a confidence level of 95% (k = 1.96).

A4.8. REPORTING OF RESULTS

The sample was submitted by the client to the laboratory and sampling uncertainty is not included in the expanded uncertainty.

Table 3: Tensile Test Results on Steel Bar

Measurement	**Results**
Average Width, w (mm)	12.563
Average Thickness, t (mm)	5.130
Cross-sectional Area, A (mm^2)	64.45
Maximum Load, F (N)	48598
Tensile Strength, σ (Nmm^{-2}) – reported with Measurement Uncertainty	**754 ± 9***
Tensile Strength, σ (Nmm^{-2}) – reported according to ASTM A370-02	**755****

* The expanded uncertainty associated with the result is ± 9 Nmm^{-2} at a confidence level of 95% with coverage factor of k = 1.96.

** ASTM A370-02 recommends reporting of tensile strength results between 500 and 1000 Nmm^{-2} to the nearest 5 Nmm^{-2}. Hence this result reflects rounding to the nearest 5 Nmm^{-2}.

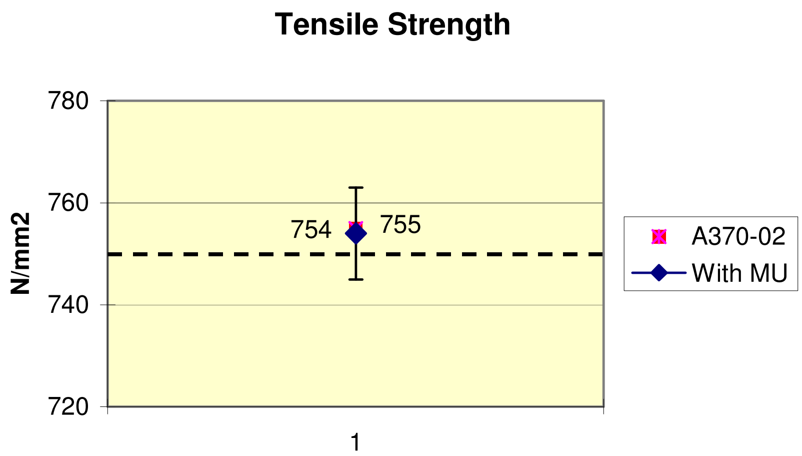
A4.9. ASSESSMENT OF COMPLIANCE WITH SPECIFICATION

The results obtained are summarised in Table 4.

Table 4: Summary of Test Results On Steel Sample

	Steel Sample	
	ASTM A370-02	**With MU**
Tensile Strength (Nmm^{-2})	755	754
Expanded Uncertainty at 95% CI	-	9
Tensile Strength Range (Nmm^{-2})	-	745 - 763
Sample Specifications (Nmm^{-2})	Min 750	Min 750

The graphical representation of the result is reflected in Figure 1.



`

Fig. 1 Graphical Representation of Tensile Strength Results

The measured result is within the specification limit, but by a margin less than half of the uncertainty interval; it is therefore not possible to state compliance based on 95% level of confidence. However the result indicates compliance is more probable than non-compliance.

ANNEX

UNCERTAINTY BUDGET TABLE

Source of Uncertainty	Symbol	Type	Uncertainty Value	Probability Distribution	Coverage Factor	Standard Uncertainty $u(x_i)$	Sensitivity Coefficient c_i	$\lvert c_i \rvert$ X $u(x_i)$	Degree of Freedom v
Tensile Machine	u_F	B	1% of Force	Rectangular	√3	280N	0.01552	4.3	∞
Micrometer (Width, Thickness)	$u_{micrometer}$	B	0.003mm	Rectangular	√3	0.0017mm	-	-	∞
Operator Error (Width, Thickness)	$u_{operator}$	B	0.005mm	Rectangular	√3	0.0029mm	-	-	∞
Sample Preparation - Width	u_{sample}	A	0.015mm	-	-	0.0087mm	-	-	2
Sample Preparation - Thickness	u_{sample}	A	0.020mm	-	-	0.012mm	-	-	2
Combined Standard Uncertainty of Width	U_w	-	-	-	-	0.0093mm	-60.02	0.56	2.6
Combined Standard Uncertainty of Thickness	U_t	-	-	-	-	0.012mm	-147.0	1.76	2.0
Combined Standard Uncertainty of Tensile Strength	u_c	-	-	-	-	4.7Nmm^{-2}	-	-	∞
Expanded Standard Uncertainty of Tensile Strength	U_{exp}	-	9.2Nmm^{-2}	-	1.96	-	-	-	-

EXAMPLE (A5) – DENSITY OF HARDENED CONCRETE CUBE, MEASUREMENT UNCERTAINTY ESTIMATED USING NUMERICAL DIFFERENTIATION

A5.1 INTRODUCTION

This example serves to illustrate the estimation of measurement uncertainty according to Guide to the Expression of Uncertainty in Measurement (GUM: 1995) in the determination of density of a single hardened concrete cube tested to SS78: Part A14: 1987 using measured dimensions.

Description of test: In this test the density of a block of concrete in the shape of a cube is determined. The mass of the cube is estimated by repeated weighing and the volume calculated by multiplying its average size obtained from repeated measurement of its orthogonal dimensions. The density, ρ , is computed from the average weight divided by the volume.

The working uses estimated uncertainties derived from equipment calibration and repeated observations.

A5.2 MODEL

Density of cubes, $\rho = \dfrac{M}{V}$ …. (1)

where M = mass of cube, and

V = volume of the cube,

$= L_{avg}^{3}$ …. (2)

L_{avg} = average of 3 pairs of orthogonal dimensions

A5.3 MEASUREMENT

Mass, M (kg)			Orthogonal dimensions (m)						Density
Repeated weighing			pair #1		pair #2		pair #3		ρ
8.5176	8.5177	8.5176	0.15110	0.15018	0.14981	0.14991	0.14865	0.14932	(kg/m^3)
Average, M = 8.51763 Std Dev = 5.77348×10^{-5}			Average, L_{avg} = 0.149828 Std Dev = 8.23976×10^{-4}						2532.43
			Volume, $V = L_{avg}^{3}$ = 3.36343×10^{-3} m^3						

A5.4 SOURCES OF UNCERTAINTY

Thus the major sources of uncertainties are:

a) Mass measurement
- i) Accuracy of measured value,
  Calibration of balance
- ii) Readability of machine output,
  Resolution of display
- iii) Repeatability of results
  Statistical representativeness of reading

b) Dimension measurement
- i) Accuracy of measured value,
  Calibration of vernier
- ii) Readability of machine output,
  Resolution of scale
- iii) Repeatability of results
  Statistical representativeness of reading

c) Sampling
- Only one specimen was received and tested
  Thus sampling uncertainty need not be considered

A5.5 ESTIMATION OF STANDARD UNCERTAINTY FROM MAJOR COMPONENTS

a) Mass measurement:

a. i) Accuracy of measured value,
From calibration report, "The estimated uncertainty ±0.1g at confidence level of approximately 95% (k=1.96)"

Standard uncertainty, $u_{M,v}$ $= \frac{0.1}{1.96}$ = 0.051g = 5.1×10^{-5}kg

Type B evaluation, based confidence level of 95%, k=1.96
Degree of freedom, $\nu_{M,v}$ = $\propto$

a. ii) Readability of machine output,
For digital scale, the resolution / fluctuation of the indicator is used, refer ISO 7500. Resolution of digital indicator is 0.1g assume rectangular dist.

Standard uncertainty, $u_{M,o}$ $= \frac{0.1}{\sqrt{3}}$ = 0.058g = 5.8×10^{-5}kg

Type B evaluation,
Degree of freedom, $\nu_{M,o}$ = $\propto$

a. iii) Repeatability of results

Mass, M (kg)				
Repeated weighing			Average	Std Dev
8.5176	8.5177	8.5176	8.51763	5.77348×10^{-5}

Standard uncertainty, $u_{M,r}$ $= \frac{5.77348\times10^{-5}}{\sqrt{3}}$ = 3.33332×10^{-5}kg

Type A evaluation
Degree of freedom, $\nu_{M,r}$ = $n-1$ = 3-1 = 2

Therefore the combined standard uncertainty of the mass measurement, $u_{M,}$

$$u_M = \sqrt{u_{M,v}{}^2 + u_{M,o}{}^2 + u_{M,r}{}^2} = \sqrt{(5.1\times10^{-5})^2 + (5.8\times10^{-5})^2 + (3.33332\times10^{-5})^2}$$

$= 8.4\times10^{-5}$ kg

b) Dimension measurement

b. i) Accuracy of measured value,
From calibration report, the estimated uncertainty is ± 0.01mm at confidence level of approximately 95% (k=2)".

i.e., Standard uncertainty, $u_{L,v}$ $= \frac{0.01}{2}$ = 0.0050mm = 5.0×10^{-6}m

Type B evaluation, based confidence level of 95%, k=2
Degree of freedom, $\nu_{L,v}$ = 50

b. ii) Readability of machine output,
For analogue scale, the resolution of the instrument is used. Resolution is 0.01mm assume rectangular distribution.

Standard uncertainty, $u_{L,o}$ = $\frac{0.01/2}{\sqrt{3}}$ = 0.0029mm = 2.9×10^{-6}m

Type B evaluation, based on grading of vernier caliper scale

Degree of freedom, $\nu_{L,o}$ = ∞

b. iii) Repeatability of results

Orthogonal dimensions (m)					
pair #1		Pair #2		pair #3	
0.15110	0.15018	0.14981	0.14991	0.14865	0.14932
Average = 0.149828			Std Dev = 8.23976×10^{-4}		

Standard uncertainty, $u_{L,r}$ = $\frac{8.23976\times10^{-4}}{\sqrt{6}}$ = 3.36387×10^{-4}m

Type A evaluation,

Degree of freedom, $\nu_{L,r}$ = $n-1$ = $6-1$ = 5

Therefore the combined standard uncertainty of dimension measurement, $u_{l,}$

$$u_L = \sqrt{u_{L,v}^2 + u_{L,o}^2 + u_{L,r}^2} = \sqrt{(5.0\times10^{-6})^2 + (2.9\times10^{-6})^2 + (3.36387\times10^{-4})^2}$$

= 3.36437×10^{-4}m

A5.6 COMBINED STANDARED UNCERTAINTY OF THE MAJOR COMPONENTS

The combined uncertainty of volume, calculated from $V = L_{avg}{}^3$ is calculated as follows:

Matrix of Parameters	Parameter	L_{avg} m	
	Std Uncertainty	3.36437×10^{-4}	$= u_L$
Parameter	Primary Value	0.149828	$= L_{avg}$
L_{avg}	0.149828	0.150164	$= L_{avg} + u_L$

Calculation

Volume, $V = L_{avg}{}^3$ m^3	a=3.36340×10^{-3}	b= 3.38611×10^{-3}	$= V = (L_{avg} + u_L)^3$
Difference, d m^3		2.27084×10^{-5}	= b-a
Square of d, m^3		5.15670×10^{-10}	= d^2
Sum of d^2, m^3		5.15670×10^{-10}	= $\sum d^2$
Combine. uncertainty, sq. root of sum d^2 m^3		2.27084×10^{-5}	= $\sqrt{\sum d^2}$

Therefore the combine uncertainty of volume, u_V

u_V = 2.27084×10^{-5} m^3

The combined uncertainty of density, calculated from $\rho = M/V$ is calculated as follows:

Matrix of Parameters	Parameter	M kg	V m^3	
	Std Uncertainty	8.4×10^{-5}	2.27084×10^{-5}	
Parameter	Primary Value	8.51763	3.36343×10^{-3}	
M	8.51763 kg	8.51771	8.51763	Diagonal = P+u_P
V	3.36343×10^{-3} m^3	3.36343×10^{-3}	3.38614×10^{-3}	Off diagonal = P
Calculation				
Density, $\rho = M/V$	2.53242×10^{3}	2.53245×10^{3}	2.51544×10^{3}	kg/m^3
Difference, d		2.49745×10^{-2}	-1.69831×10^{1}	kg/m^3
Square of d ,		6.23726×10^{-4}	2.88427×10^{2}	kg/m^3
Sum of d^2 ,		2.88428×10^{2}		kg/m^3
Combine. uncertainty, sq. root of sum d^2		1.69832×10^{1}		kg/m^3

Note: P is the primary value of the parameter and u_P is the standard uncertainty of the parameter

Therefore the combine uncertainty of density, u_c

u_c = 16.9832 kg/m^3

A5.7 ESTIMATE OF EXPANDED UNCERTAINTY

Coverage factor,

Without the computing partial differentiation, the sensitivity coefficient, c_i, is estimated assuming a linear relation between u and u_{ii}.

Effective degree of freedom of volume, ν_V

$$\nu_V = \frac{u^4}{\sum_{\substack{i=1\\j=1}}^{N} \frac{c_i^{\,4} u_{i,j}^{\,4}}{\nu_{i,j}}} = \frac{u^4}{\sum_{\substack{i=1\\j=1}}^{N} \left[\frac{\left(\frac{u}{u_{ji}}\right)^4 u_i^{\,4}}{\nu_i} \right]} = \frac{u_v^{\,4}}{\sum_{i=1}^{N} \left[\frac{\left(\frac{u_v}{u_L}\right)^4 u_{L,i}^{\,4}}{\nu_{L,i}} \right]} = \frac{1}{\sum_{i=1}^{N} \left[\frac{\left(\frac{u_{L,i}}{u_L}\right)^4}{\nu_{L,i}} \right]}$$

$$= \frac{\left(2.27084\times10^{-5}\right)^4}{\left(\frac{\left(\frac{2.27084\times10^{-5}}{3.36437\times10^{-4}}\right)^4 \times \left(5.0\times10^{-6}\right)^4}{50} + \frac{\left(\frac{2.27084\times10^{-5}}{3.36437\times10^{-4}}\right)^4 \times \left(2.9\times10^{-6}\right)^4}{\propto} + \frac{\left(\frac{2.27084\times10^{-5}}{3.36437\times10^{-4}}\right)^4 \times \left(3.36387\times10^{-4}\right)^4}{5} \right)}$$

$$= \frac{1}{\left(\frac{\left(\frac{5.0\times10^{-6}}{3.36437\times10^{-4}}\right)^4}{50} + \frac{\left(\frac{2.9\times10^{-6}}{3.36437\times10^{-4}}\right)^4}{\propto} + \frac{\left(\frac{3.36387\times10^{-4}}{3.36437\times10^{-4}}\right)^4}{5} \right)}$$

= 5

Effective degree of freedom of mass, ν_M

$$\nu_M = \frac{u^4}{\sum_{\substack{i=1\\j=1}}^{N} \frac{c_i^4 u_{i,j}^4}{\nu_{i,j}}} = \frac{u^4}{\sum_{\substack{i=1\\j=1}}^{N} \left[\frac{\left(\frac{u}{u_{ji}}\right)^4 u_i^4}{\nu_i} \right]} = \frac{u_M^4}{\sum_{i=1}^{N} \left[\frac{\left(\frac{u_M}{u_M}\right)^4 u_{M,i}^4}{\nu_{M,i}} \right]} = \frac{u_M^4}{\sum_{i=1}^{N} \left[\frac{u_{M,i}^4}{\nu_{M,i}} \right]}$$

$$= \frac{\left(8.4\times10^{-5}\right)^4}{\left(\frac{\left(5.1\times10^{-5}\right)^4}{\propto} + \frac{\left(5.8\times10^{-5}\right)^4}{\propto} + \frac{\left(3.33332\times10^{-5}\right)^4}{2} \right)}$$

= 81

Therefore the effective degree of freedom of density, ν_{eff}

$$\nu_{eff} = \frac{u^4}{\sum_{\substack{i=1\\j=1}}^{N} \frac{c_i^4 u_{i,j}^4}{\nu_{i,j}}} = \frac{u^4}{\sum_{\substack{i=1\\j=1}}^{N} \left[\frac{\left(\frac{u}{u_{ji}}\right)^4 u_i^4}{\nu_i} \right]} = \frac{u_c^4}{\sum_{\substack{i=1\\j=1}}^{N} \left[\frac{\left(\frac{u_c}{u_{ji}}\right)^4 u_i^4}{\nu_i} \right]} = \frac{u_c^4}{\frac{\left(\frac{u_c}{u_M}\right)^4 u_M^4}{\nu_M} + \frac{\left(\frac{u_c}{u_{ji}}\right)^4 u_i^4}{\nu_V}}$$

$$= \frac{1}{\frac{1}{\nu_M} + \frac{1}{\nu_V}}$$

$$= \frac{1}{\frac{1}{81} + \frac{1}{5}}$$

= 5

k = 2.57 at 95% level of confidence

Expanded uncertainty, $U = k\, u_c$

$$= 2.57 \times 16.9832 \text{ kg/m}^3 = 43.6 \text{ kg/m}^3$$

Therefore, the density of the hardened concrete cube $= 2532.4 \pm 43.6$ kg/m^3

A5.8 REPORTING OF RESULTS

Density of the hardened concrete cube, $\rho = 2532.4 \pm 43.6$ kg/m^3 at level of confidence of 95% (k=2.57)

Therefore, the density of the hardened concrete cube is 2530 kg/m^3 tested in accordance to SS78: Part A14:1987.

The test method requires rounding to nearest 10 kg/m^3, without quoting the measurement uncertainty.

APPENDIX B

ISO/TS 21748 APPROACH

B1. Introduction

B1.1 Technical Specification ISO/TS 21748 provides an appropriate methodology for estimating uncertainty associated with results of a wide range of standard test methods subjected to collaborative study in accordance with ISO 5725-2. The methodology complies fully with the relevant principles of the GUM, whilst taking into account the method performance data obtained by collaborative study.

B1.2 The general approach used in this Technical Specification requires that

- Estimates of the repeatability, reproducibility and trueness of the method in use, obtained by collaborative study as described in ISO 5725-2, be available from published information about the test method in use. These provide estimates of the intra- and inter-laboratory components of variance, together with an estimate of uncertainty associated with the trueness of the method;

- The laboratory confirms that its implementation of the test method is consistent with the established performance of the test method by checking its own bias and precision. This confirms that the published data are applicable to the results obtained by the laboratory;

- Any influences on the measurement results that were not adequately covered by the collaborative study be identified and the variance associated with the results that could arise from these effects be quantified.

B1.3 An uncertainty estimate is made by combining the relevant variance estimates in the manner prescribed by GUM.

B1.4 The ISO/TS 21748 assumes that recognised, non-negligible systematic effects are corrected, either by applying a numerical correction as part of the method of measurement, or by investigation and removal of the cause of the effect.

B2. General Principles

B2.1 Individual Results and Measurement Process Performance

B2.1.1 Measurement uncertainty relates to individual results. Repeatability, reproducibility, and bias, by contrast relate to the performance of a measurement or testing process.

B2.1.2 The ISO/TS 21748 requires that process performance figures derived from method-performance studies are relevant to all individual measurement results produced by the process. It will be seen that this condition requires supporting evidence in the form of appropriate quality control and assurance data for the measurement process.

B2.1.3 It should also be noted that difference between individual test items may additionally need to be taken into account. However, it is unnecessary to undertake individual and detailed uncertainty studies for every test item for a well characterised and stable measurement process.

B2.2 Applicability of Reproducibility Data

B2.2.1 The application of the principles of the ISO/TS 21748 is based on two principles

- First, the reproducibility standard deviation obtained in a collaborative study is a valid basis for measurement uncertainty evaluation;

- Second, effects not observed within the context of the collaborative study must be demonstrably negligible or explicitly allowed for. The latter principle is implemented by an extension of the basic model used for collaborative study;

B2.3 Basic Equation for the Statistical Model

B2.3.1 The statistical model on which this Technical Guide is based is formulated in Equation (1):

$$y = \mu + \delta + B + \sum c_i x'_i + e \qquad \ldots..(1)$$

where

y is an observed result, assumed to be calculated from the equation:
$$y = f(x_1, x_2, \ldots, x_n);$$

μ is the (unknown) expectation of ideal results;

δ is a term representing bias intrinsic to the measurement method;

B is the laboratory component of bias;

x'_i is the deviation from the nominal value of x_i;

c_i is the sensitivity coefficient, equal to $\delta y/\delta x_i$;

e is the residual error term.

B and e are assumed normally distributed, with expectation zero and a variance of σ_L^2 and σ_r^2, respectively. These terms form the model used in ISO 5725-2 for the analysis of collaborative study data.

Since the observed standard deviations of method bias, δ, laboratory bias, B, and residual error, e, are overall measures of dispersion under the conditions of the collaborative study, the summation $\sum c_i x'_i$ is over those effects subject to deviation other than those incorporated in δ, B, or e, and the summation accordingly provides a method for incorporating effects of operations that are not carried out in the course of a collaborative study.

Examples of such operations include the following:

a) preparation of test item carried out in practice for each test item, but carried out prior to circulation in the case of the collaborative study;

b) effects of sub-sampling in practice when test items subjected to collaborative study were, as is common, homogenised prior to the study.

B2.3.2 Given the model described by Equation (1), the uncertainty $u(y)$.associated with an observation can be estimated using Equation (2)

$$u^2(y) = u^2(\hat{}\delta) + s_L^2 + \sum c_i^2 u^2(x_i) + s_r^2 \qquad \ldots..(2)$$

where

s_L^2 s the estimated variance of B;

s_r^2 is the estimated variance of e;

$u(\hat{}\delta)$ is the uncertainty associated with δ due to the uncertainty of estimating δ by measuring a reference measurement standard or reference material with certified value $\hat{}\delta$;

$u(x_i)$ is the uncertainty associated with x'_i.

Given that the reproducibility standard deviation s_R is given by $s_R^2 = s_L^2 + s_r^2$, s_R^2 can be substituted for $s_L^2 + s_r^2$ and Equation (2) reduces to Equation (3)

$$u^2(y) = u^2(\hat{\delta}) + s_R^2 + \sum c_i^2 u^2(x_i) \qquad \ldots..(3)$$

B2.4 Repeatability Data

B2.4.1 It will be seen that repeatability data are used in the ISO/TS 21748 primarily as a check on precision, which, in conjunction with other tests, confirms that a particular laboratory may apply reproducibility and trueness data in its estimates of uncertainty. Repeatability data are also employed in the calculation of the reproducibility component of uncertainty.

B3. Evaluating Uncertainty Using Repeatability, Reproducibility and Trueness Estimates

B3.1 Procedure for Evaluating Measurement Uncertainty

B3.1.1 The principles on which the ISO/TS 21748 is based, lead to the following procedure for evaluating measurement uncertainty.

Step	Description	Symbols
1.	Obtain from published information or assessment of the method, estimates of a. Repeatability b. Reproducibility c. Trueness	s_R – estimated reproducibility standard deviation $u(\hat{\delta})$ – uncertainty associated with δ due to the uncertainty of estimating δ by measuring a reference measurement standard or reference material with certified value (related to bias)
2.	Check lab bias for the measurements is within that expected on the basis of data from Step 1	s_L – experimental or estimated inter-laboratory standard deviation
3.	Check precision of current measurements is within that expected on the basis of the repeatability and reproducibility estimates from Step 1	The measure of precision is usually expressed in terms of imprecision and computed as a standard deviation of test results. Less precision is reflected by a higher standard deviation
4.	Any influences not adequately covered in references for Step 1 must be quantified based on Variance and Sensitivity Coefficient	$\sum c_i^2 u^2(x_i)$
5.	Combine Reproducibility Estimate, Uncertainty Associated with Trueness and the effects of Any Additional Influences, if all the above checks are acceptable	

B3.2 Differences Between Expected and Actual Precision

B3.2.1 Where the precision differs in practice from that expected from the studies in Step 1, the associated contributions to uncertainty should be adjusted, eg adjustments to reproducibility estimates for the common case where the precision is approximately proportional to level of response.

B4. Establishing the Relevance of Method Performance Data to Measurement Results for a Particular Measurement Process

B4.1 General

The results of collaborative study yield performance indicators (s_R, s_r) and, in some circumstances a method bias estimate, which form a "specification" for the method performance. In adopting the method for its specified purpose, a laboratory is normally expected to demonstrate that it is meeting this "specification". In most cases, this is achieved by studies intended to verify control of repeatability (see B4.3) and of the laboratory component of bias (see B4.2), and by continued performance checks [quality control and assurance (see B4.4)].

B4.2 Demonstrating Control of the Laboratory Component of Bias

B4.2.1 General Requirements

B4.2.1.1 A laboratory should demonstrate, in its implementation of a method, that bias is under control, that is, the laboratory component of bias is within the range expected from the collaborative study.

B4.2.1.2 In general, a check on the laboratory component of bias constitutes a comparison between laboratory results and some reference value(s), and constitutes an estimate of B. Equation (2) shows that the uncertainty associated with variations in B is represented by s_L, itself included in s_R. However, because the bias check is itself uncertain, the uncertainty of the comparison in principle increases the uncertainty of the results in future applications of the method. For this reason, it is important to ensure that the uncertainty associated with the bias check is small compared to s_R, (ideally less than 0.2 s_R).

B4.2.2 Methods of Demonstrating Control of the Laboratory Component of Bias

B4.2.2.1 General

Bias control may be demonstrated by various methods, examples of which are detailed in the ISO/TS 21748

a) Study of certified reference material or measurement standard
b) Comparison with a definitive test method of known uncertainty
c) Comparison with other laboratories using the same method

B4.2.3 Detection of Significant Laboratory Component of Bias

The ISO/TS 21748 is applicable only where the laboratory component of bias is demonstrably under control. Where excessive bias is detected, it is assumed that action will be taken to bring the bias within the required range before proceeding with measurements.

B4.3 Verification of Repeatability

B4.3.1 The test laboratory should show that its repeatability is consistent with the repeatability standard deviation obtained in the course of the collaborative exercise. The demonstration of repeatability should be achieved by replicate analysis of one or more suitable test materials, to obtain (by pooling results if necessary) a repeatability standard deviation s_i with v_i degrees of freedom. The values of s_i should be compared, using a F-test at the 95% level of confidence if necessary, with the repeatability standard deviation s_r derived from the collaborative study.

B4.3.2 If s_i is found to be significantly greater than s_r, the laboratory concerned should either identify and correct the causes or use s_i in place of s_r in all uncertainty estimates calculated using ISO/TS 21748. The detailed treatment is presented in the ISO/TS 21748.

B4.4 Continued Verification of Performance

In addition to preliminary estimation of bias and precision, the laboratory should take due measures to ensure that the measurement procedure remains in a state of statistical control. In particular, this will involve the following:

a) appropriate quality control, including regular checks on bias and precision.

b) quality assurance measures, including the use of appropriately trained and qualified staff operating within a suitable quality system.

B5. Establishing Relevance to the Test Item

B5.1 General

In a collaborative study or an estimation of intermediate measures of precision under Parts 2 and 3 of ISO 5725, it is normal to measure values on homogeneous materials or items of a small number of types. It is also common practice to distribute prepared materials. Routine test items, on the other hand, may vary widely, and may require additional treatment prior to testing. For example, environmental test samples are frequently supplied dried, finely powdered and homogenized for collaborative study purposes; routine test samples are wet, inhomogeneous and coarsely divided. It is accordingly necessary to investigate, and if necessary allow for, these differences.

B5.2 Sampling

B5.2.1 Inclusion of Sampling Process

Collaborative studies rarely include a sampling step; if the method used in-house involves sub-sampling, or the procedure as used routinely is estimating a bulk property from a small sample, then the effects of sampling should be investigated.

B5.2.2 Inhomogeneity

Where test materials are found to be significantly inhomogeneous, the variance estimate from homogeneity studies should be converted directly to a standard uncertainty.

B5.3 Sample Preparation and Pre-Treatment

In most studies, samples are homogenised, and may additionally be stabilised, before distribution. It may be necessary to investigate and allow for the effects of the particular pre-treatment procedures applied in-house. Typically, such investigations establish the effect of the procedure on the measurement result by studies on materials with approximately or accurately established properties.

B5.4 Changes in Test-Item Type

The uncertainty arising from changes in type or composition of test items compared to those used in the collaborative study should, where relevant, be investigated.

B5.5 Variation of Uncertainty with Level of Response

B5.5.1 Adjusting s_R

It is common to find that some or most contributions to uncertainty for a given measurement are dependent on the value of the measurand. ISO 5725-2 considers three simple cases where the reproducibility standard deviation for a particular positive value m is approximately described by one of the models

$$\hat{s}_R = bm$$

$\hat{s}_R = a + bm$

$\hat{s}_R = cm^d$

where

$\hat{s}_R$ is the adjusted reproducibility standard deviation calculated from the approximate model;

a, b, c and d are empirical coefficients derived from a study of five or more different test items with different mean responses m (a, b and c are positive)

Where one of the above equations applies, the uncertainty should be based on a reproducibility estimate calculated using the appropriate model.

Where the provisions of B4.3 apply, $\hat{s}_R$ should also reflect the changed contribution of the repeatability term s_r.

B5.5.2 Changes in other Contributions to Uncertainty

In general, where any contribution to uncertainty changes with the measured response in a predictable manner, the relevant standard uncertainty in y should be adjusted accordingly.

B6. Additional Factors

Clause B5 considers the main factors that are likely to change between collaborative study and routine testing. It is possible that other effects may operate in particular instances. Where these effects are not negligible, the uncertainty associated with such factors should be estimated, recorded and combined with other contributions in the normal way [ie following the summation principle in Equation (3)].

B7. General Expression for Combined Standard Uncertainty

Equation (3), taking into account the need to use the adjusted estimate $\hat{s}_R^2$ instead of s_R^2 to allow for factors discussed in clause B5, leads to the general expression in Equation (4) for the estimation of the combined standard uncertainty $u(y)$associated with a result y:

$$u^2(y) = \hat{s}_R^2 + u^2(\hat{\delta}) + \sum_{i=1,n'} [c_i^2 u^2(x_i)] \qquad \text{.....(4)}$$

where $u(\hat{\delta})$ is calculated as specified below:

$$u(\hat{\delta}) = [\, s_R^2 - (1-1/n).s_r^2 / p\,]^{0.5}$$

where

p is the number of laboratories

n is the number of replicates in each laboratory

The variable $u(B)$ does not appear in Equation (4) because s_L, the uncertainty associated with B, is already included in $\hat{s}_R^2$. The subscript "i" covers effects identified in clause B5 and B6.

Clearly, where and effects and uncertainties are small compared to s_R, they may, for most practical purposes, be neglected.

WORKED EXAMPLES – BASED ON ISO/TS 21748 APPROACH

The following generic worked examples are intended to show how the principles in this Technical Guide can be applied to the tests in the civil engineering and mechanical testing fields.

Example (B1) - Compressive Strength of Hardened Concrete Cubes

Example (B2) - Concrete Non-Destructive Testing - Ultrasonic Pulse Velocity Test on Wall and Slab

Example (B3) - Concrete Non-Destructive Testing - Windsor Probe Test on Wall

Example (B4) - Concrete Non-Destructive Testing - Rebound Hammer Test on Wall and Slab

Example (B5) - Rockwell ‘C’ Hardness Test on Metallic Sample

EXAMPLE (B1) - COMPRESSIVE STRENGTH OF HARDENED CONCRETE CUBES

B1.1. Introduction

This example serves to illustrate the estimation of measurement uncertainty according to ISO/TS 21748 approach. Repeatability and reproducibility information as required by the approach are obtained from proficiency testing data and calculated based on ISO 5725-2 (shown below from step B1.2 to step B1.6). The proficiency testing data is obtained from compressive strength of cubes tested in accordance to SS78: Part A16: 1987.

Definition*:*

i	laboratory or operator	x	data set using mean
j	level, e.g. targeted strength of cubes	y	individual results
k	individual results by i	$\bar{y}$	mean of y of the laboratory or operator
p	no. of laboratory or operator	$\bar{\bar{y}}$	general mean of the level
q	no. of levels	s	spread of results, e.g. standard deviation
n	no. of results by each laboratory or operator		

B1.2. Original Data

Laboratory	Level	
	50MPa	…..j….q
Lab 1	51.5	
	49.0	
	51.0	
	49.5	
	50.0	
	45.5	
Lab 2	51.0	
	49.0	
	49.0	
	50.0	
	51.0	
	52.5	
Lab 3	49.5	
	49.5	
	48.0	
	50.0	
	50.5	
	50.0	
Lab 4	50.5	
	50.0	
	48.5	
	49.5	
	46.0	
	50.0	
Lab 5	53.0	
	52.5	
	54.0	
	50.5	
	53.0	
	51.0	

Lab 6	50.0	
	47.0	
	47.5	
	47.5	
	50.0	
	50.0	
Lab 7	51.0	
	51.5	
	52.0	
	50.0	
	50.0	
	49.0	
Lab 8	46.0	
	51.5	
	46.5	
	51.0	
	52.0	
	50.0	
Lab 9	49.0	
	52.0	
	49.0	
	47.5	
	48.5	
	51.0	
. . i . p		y_{ij1} , k=1 … y_{ijk} … $y_{ijn_{ij}}$, k=n_{ij}

B1.3. Mean for each Laboratory or Operator

Submission Lab or Operator	Level		
	50MPa		…..j….q
	$\overline{y_{ij}}$	n_{ij}	
Lab 1	49.42	6	
Lab 2	50.42	6	
Lab 3	49.58	6	
Lab 4	49.08	6	
Lab 5	52.33	6	
Lab 6	48.67	6	
Lab 7	50.58	6	
Lab 8	49.50	6	
Lab 9	49.50	6	
. i . p			$\overline{y_{ij}} = \frac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} y_{ijk}$
General mean, $\widehat{m}_j = \overline{\overline{y_j}}$	49.90		$\widehat{m}_j = \overline{\overline{y_j}} = \frac{\sum_{i=1}^{p_j} \left(n_{ij} \times \overline{y_{ij}}\right)}{\sum_{i=1}^{p_j} n_{ij}}$

Note: Express to 1 more significant digit than original data

B1.4. Spread of each Cell (commonly sample standard deviation)

Submission Lab or Operator	Level		
	50MPa		…..j….q
	S_{ij}	S_{ij}^{2}	
Lab 1	2.131	4.542	
Lab 2	1.357	1.842	
Lab 3	0.8612	0.7417	
Lab 4	1.656	2.742	
Lab 5	1.329	1.767	
Lab 6	1.472	2.167	
Lab 7	1.114	1.242	
Lab 8	2.608	6.800	
Lab 9	1.673	2.800	
. i . p			$S_{ij} = \sqrt{\frac{1}{n_{ij}-1}\sum_{k=1}^{n_{ij}}\left(y_{ijk} - \overline{y_{ij}}\right)^2}$
$\sum_{i=1}^{p} S_{ij}^{2}$		24.64	

Note: Express to 1 more significant digit than original data

B1.5. Scrutiny of Results for Consistency and Outliers

- Graphical Method

Mandel's h statistics, for between-laboratory / between-operator consistency

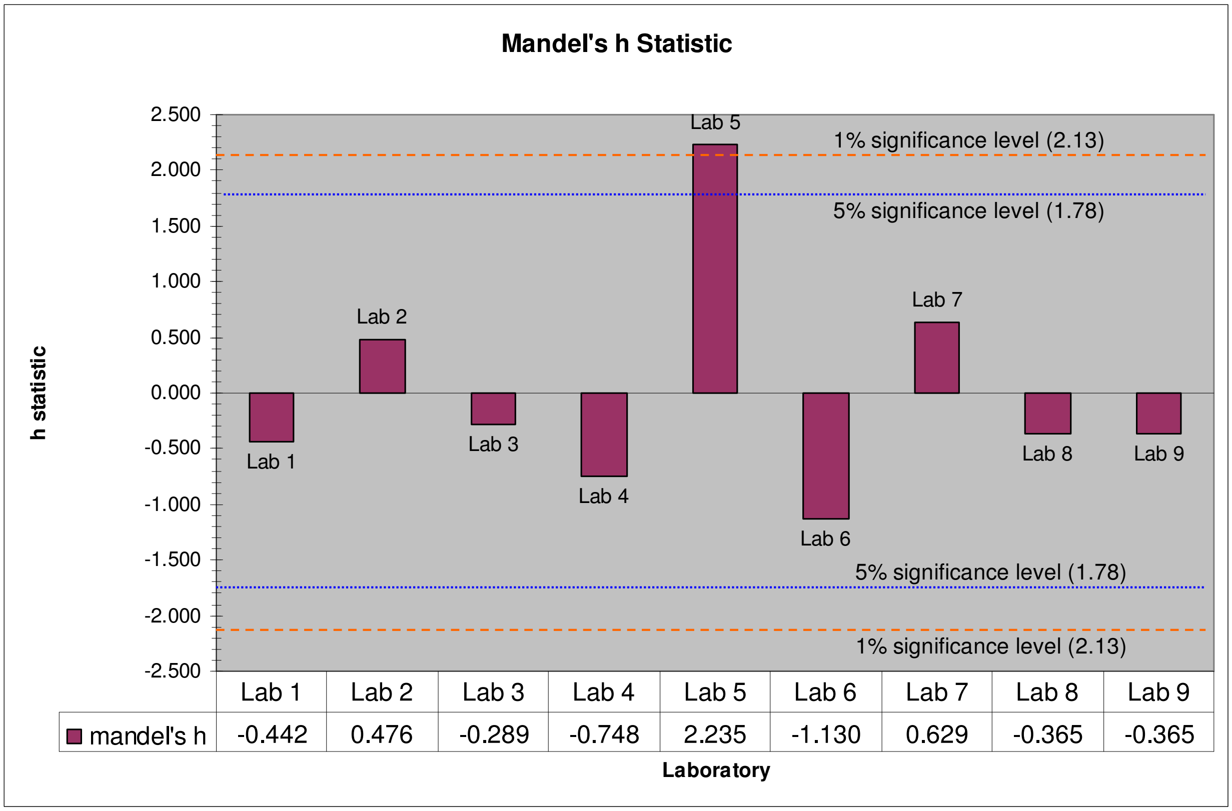
Submission Lab or Operator	Level	
	50MPa	…..j….q
Lab 1	-0.4419	
Lab 2	0.4759	
Lab 3	-0.2889	
Lab 4	-0.7478	
Lab 5	2.235 **	Outlier
Lab 6	-1.130	
Lab 7	0.6289	
Lab 8	-0.3654	
Lab 9	-0.3654	
. i . p		$h_{ij} = \frac{\overline{y_{ij}} - \overline{\overline{y_j}}}{\sqrt{\frac{1}{p_j-1}\sum_{i=1}^{p_j}\left(\overline{y_{ij}} - \overline{\overline{y_j}}\right)^2}}$
p_j	9	n_j = no. of test results occurring in majority of the cells
n_j	6	
1% significance level	2.13	
5% significance level	1.78	

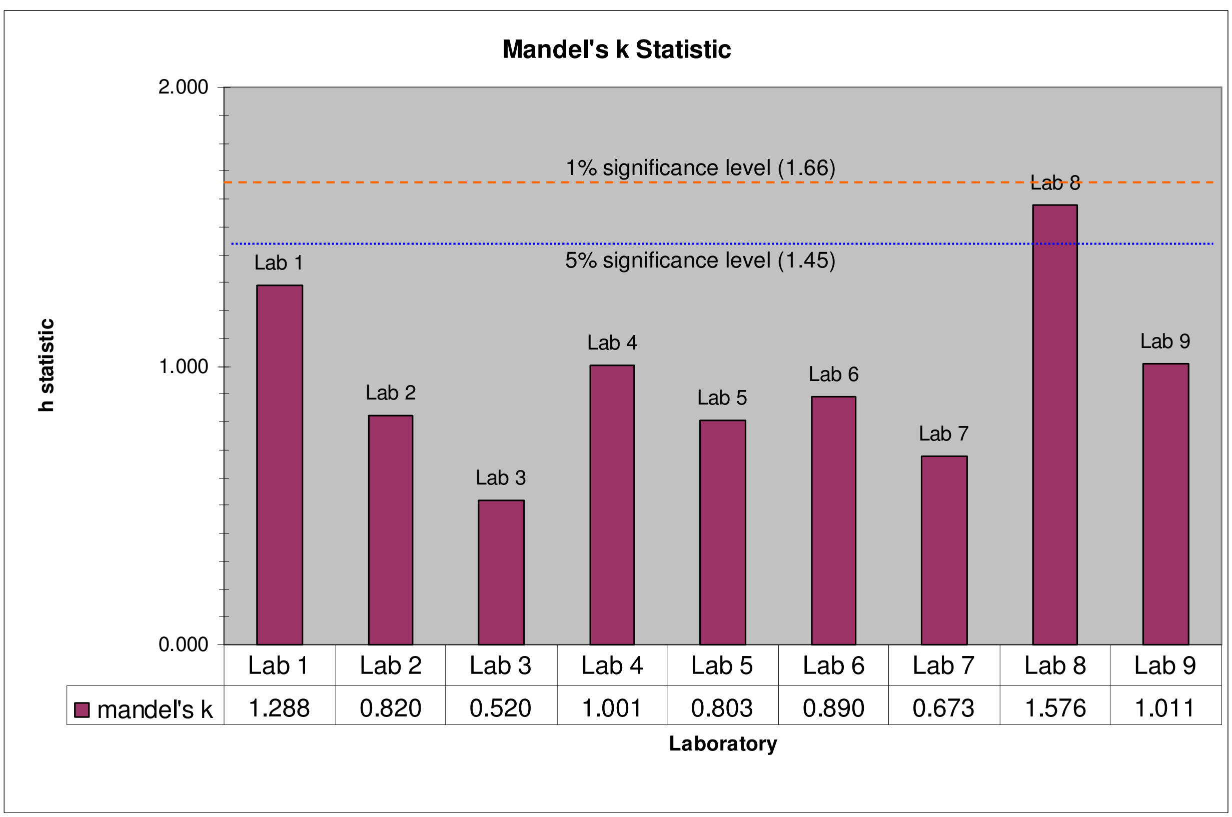
Mandel's k statistics, for within-laboratory / operator consistency

<table>
<tr><th rowspan="2">Submission
Lab or Operator</th><th colspan="2">Level</th></tr>
<tr><th>50MPa</th><th>…..j….q</th></tr>
<tr><td>Lab 1</td><td>1.288</td><td></td></tr>
<tr><td>Lab 2</td><td>0.8201</td><td></td></tr>
<tr><td>Lab 3</td><td>0.5205</td><td></td></tr>
<tr><td>Lab 4</td><td>1.000</td><td></td></tr>
<tr><td>Lab 5</td><td>0.8033</td><td></td></tr>
<tr><td>Lab 6</td><td>0.8896</td><td></td></tr>
<tr><td>Lab 7</td><td>0.6734</td><td></td></tr>
<tr><td>Lab 8</td><td>1.576 *</td><td>Straggler</td></tr>
<tr><td>Lab 9</td><td>1.011</td><td></td></tr>
<tr><td>.
i
.
p</td><td></td><td>$k_{ij} = \frac{S_{ij}\sqrt{p_j}}{\sqrt{\sum_{i=1}^{p_j} S_{ij}^2}}$</td></tr>
<tr><td>p_j</td><td>9</td><td rowspan="2">n_j = no. of test results occurring in majority of the cells</td></tr>
<tr><td>n_j</td><td>6</td></tr>
<tr><td>1% significance level</td><td>1.66</td><td></td></tr>
<tr><td>5% significance level</td><td>1.45</td><td></td></tr>
</table>

Mandel's h Test: Outlier at Level 50MPa, Lab 5.

Mandel's k Test: Straggler at Level 50MPa, Lab 8

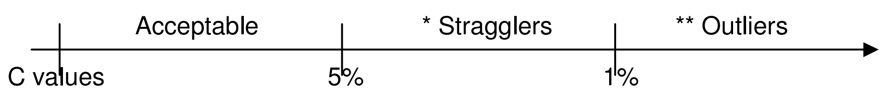

Mandel's h Statistic
h statistic
2.500
2.000
1.500
1.000
0.500
0.000
-0.500
-1.000
-1.500
-2.000
-2.500
1% significance level (2.13)
5% significance level (1.78)
5% significance level (1.78)
1% significance level (2.13)
Lab 1 Lab 2 Lab 3 Lab 4 Lab 5 Lab 6 Lab 7 Lab 8 Lab 9
mandel's h -0.442 0.476 -0.289 -0.748 2.235 -1.130 0.629 -0.365 -0.365
Laboratory


Mandel's k Statistic
h statistic
2.000
1.000
0.000
1% significance level (1.66)
5% significance level (1.45)
Lab 1 Lab 2 Lab 3 Lab 4 Lab 5 Lab 6 Lab 7 Lab 8 Lab 9
mandel's k 1.288 0.820 0.520 1.001 0.803 0.890 0.673 1.576 1.011
Laboratory

- Numerical Method

Cochran’s test, for within laboratory / operator consistency. Test only the highest spread.

Submission Lab or Operator	**Level**		
	50MPa		…..j….q
	$S_{ij,\,max}$	C	
Lab 1			
Lab 2			
Lab 3			
Lab 4			
Lab 5			
Lab 6			
Lab 7			
Lab 8	2.608	0.2760	**Acceptable**
Lab 9			
. i . p			$C = \frac{S^2_{ij,\,max}}{\sum_{i=1}^{p_j} S_{ij}^2}$
p_j	9		
n_j	6		
Critical 1%	0.423		
Critical 5%	0.360		



Cochran’s test: Lab 8 results is acceptable.

Grubb’s Test

Grubb’s Test: Check single high,
Check single low,
If there is no outlier, then
Check double high,
Check double low.

Grubb’s test, for between-laboratory/between-operator consistency. Test single highest mean.

Single High Observation	**Level**		
	50MPa		…..j….q
	lab/opr	$\overline{y}_{ij}$	
x_1 =	Lab 6	48.67	$\overline{x} = \frac{1}{p_j}\sum_{i=1}^{p_j} x_i$
…	Lab 4	49.08	
…	Lab 1	49.42	
…	Lab 8	49.50	
Sorted, x_i	Lab 9	49.50	
…	Lab 3	49.58	
…	Lab 2	50.42	
…	Lab 7	50.58	

x_p =	Lab 5	52.33	$S=\sqrt{\frac{1}{p_j-1}\sum_{i=1}^{p_j}\left(x_i-\bar{x}\right)^2}$
$\bar{x}$	49.90		
S	1.088		
G_p	2.235		
Critical 1%	2.387		
Critical 5%	2.215		$G_p=\frac{\left(x_p-\bar{x}\right)}{S}$

Acceptable | * Stragglers | ** Outliers

G values | 5% | 1%

Grubb's single high test: The highest mean is a straggler.

Grubb's test for between-laboratory / between-operator consistency. Test single lowest mean.

Single Low Observation	Level		
	50MPa		…..j….q
	lab/opr	$\bar{y}_{ij}$	
x_1=	Lab 6	48.67	$\bar{x}=\frac{1}{p_j}\sum_{i=1}^{p_j}x_i$
…	Lab 4	49.08	
…	Lab 1	49.42	
…	Lab 8	49.50	
Sorted, x_i	Lab 9	49.50	
…	Lab 3	49.58	
…	Lab 2	50.42	$S=\sqrt{\frac{1}{p_j-1}\sum_{i=1}^{p_j}\left(x_i-\bar{x}\right)^2}$
…	Lab 7	50.58	
x_p=	Lab 5	52.33	
$\bar{x}$	49.90		
S	1.088		
G_1	1.128		$G_1=\frac{\left(\bar{x}-x_1\right)}{S}$
Critical 1%	2.387		
Critical 5%	2.215		

Acceptable | * Stragglers | ** Outliers

G values | 5% | 1%

Grubb's single low test: All mean data acceptable.

Grubb's test, for between-laboratory / between-operator consistency. Test double highest mean.

Single High Observation	Level		
	50MPa		…..j….q
	lab/opr	$\bar{y}_{ij}$	
x_1 =	Lab 6	48.67	$\bar{x}_{p-1,p}=\frac{1}{p_j-2}\sum_{i=1}^{p_j-2}x_i$
…	Lab 4	49.08	
…	Lab 1	49.42	
…	Lab 8	49.50	
Sorted, x_i	Lab 9	49.50	$S^2_{p-1,p}=\sum_{i=1}^{p_j-2}\left(x_i-\bar{x}_{p-1,p}\right)^2$
…	Lab 3	49.58	
…	Lab 2	50.42	
…	Lab 7	-	

x_p =	Lab 5	-	$S^2_o = \sum_{i=1}^{p_j} \left(x_i - \bar{x}\right)^2$ $G_{2p} = \frac{S^2_{p-1,p}}{S^2_o}$
$\bar{x}_{p-1,p}$	49.45		
$S^2_{p-1,p}$	1.709		
S^2_o	9.476		
G_{2p}	0.1803		
Critical 1%	0.0851		
Critical 5%	0.1492		

** Outliers | * Stragglers | Acceptable

G values | 1% | 5%

Grubb's double high test: All mean data acceptable.

Grubb's test, for between-laboratory / between-operator consistency. Test double lowest mean.

Single High Observation	**Level**		
	50MPa		…..j….q
	lab/opr	$\bar{y}_{ij}$	
x_1 =	Lab 6	-	$\bar{x}_{1,2} = \frac{1}{p_j - 2} \sum_{i=3}^{p_j} x_i$ $S^2_{1,2} = \sum_{i=3}^{p_j} \left(x_i - \bar{x}_{1,2}\right)^2$ $S^2_o = \sum_{i=1}^{p_j} \left(x_i - \bar{x}\right)^2$ $G_{1,2} = \frac{S^2_{1,2}}{S^2_o}$
…	Lab 4	-	
…	Lab 1	49.42	
…	Lab 8	49.50	
Sorted, x_i	Lab 9	49.50	
…	Lab 3	49.58	
…	Lab 2	50.42	
…	Lab 7	50.58	
x_p =	Lab 5	52.33	
$\bar{x}_{1,2}$	50.19		
$S^2_{1,2}$	6.702		
S^2_o	9.476		
$G_{1,2}$	0.7073		
Critical 1%	0.0851		
Critical 5%	0.1492		

** Outliers | * Stragglers | Acceptable

G values | 1% | 5%

Grubb's double low test: All mean data acceptable.

B1.6. Calculation of Mean, Repeatability and Reproducibility

<table>
<tr><th rowspan="3">Submission Lab or Operator</th><th colspan="5">Level</th></tr>
<tr><th colspan="4">50MPa</th><th>…..j….q</th></tr>
<tr><th>$\bar{y}_{ij}$</th><th>S_{ij}</th><th>S_{ij}^2</th><th>n_{ij}</th><th></th></tr>
<tr><td>Lab 1</td><td>49.42</td><td>2.131</td><td>4.542</td><td>6</td><td></td></tr>
<tr><td>Lab 2</td><td>50.42</td><td>1.357</td><td>1.842</td><td>6</td><td></td></tr>
<tr><td>Lab 3</td><td>49.58</td><td>0.8612</td><td>0.7417</td><td>6</td><td></td></tr>
<tr><td>Lab 4</td><td>49.08</td><td>1.656</td><td>2.742</td><td>6</td><td></td></tr>
<tr><td>Lab 5</td><td>52.33</td><td>1.329</td><td>1.767</td><td>6</td><td></td></tr>
<tr><td>Lab 6</td><td>48.67</td><td>1.472</td><td>2.167</td><td>6</td><td></td></tr>
<tr><td>Lab 7</td><td>50.58</td><td>1.114</td><td>1.242</td><td>6</td><td></td></tr>
<tr><td>Lab 8</td><td>49.50</td><td>2.608</td><td>6.800</td><td>6</td><td></td></tr>
<tr><td>Lab 9</td><td>49.50</td><td>1.673</td><td>2.800</td><td>6</td><td></td></tr>
<tr><td>.
i
.
p</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>p_j</td><td colspan="4">9</td><td></td></tr>
<tr><td>General mean, $\hat{m}_j = \bar{\bar{y}}_j$</td><td colspan="4">49.90</td><td>$\hat{m}_j = \bar{\bar{y}}_j = \dfrac{\sum_{i=1}^{p_j}\left(n_{ij} \times \overline{y_{ij}}\right)}{\sum_{i=1}^{p_j} n_{ij}}$</td></tr>
<tr><td>Repeatability variance, S_{rj}^2</td><td colspan="4">2.738</td><td>$S_{rj}^2 = \dfrac{\sum_{i=1}^{p_j}\left[(n_{ij}-1)xS_{ij}^2\right]}{\sum_{i=1}^{p_j}(n_{ij}-1)}$</td></tr>
<tr><td>$\bar{\bar{n}}_j$</td><td colspan="4">6.0</td><td>$\bar{\bar{n}}_j = \dfrac{1}{p_j-1}\left(\sum_{i=1}^{p_j} n_{ij} - \dfrac{\sum_{i=1}^{p_j} n_{ij}^2}{\sum_{i=1}^{p_j} n_{ij}}\right)$</td></tr>
<tr><td>S_{dj}^2</td><td colspan="4">7.123</td><td>$S_{dj}^2 = \dfrac{1}{p_j-1}\left[\sum_{i=1}^{p_j}\left(n_{ij}\bar{y}_{ij}^2\right) - \bar{\bar{y}}_j^2 \sum_{i=1}^{p_j} n_{ij}\right]$</td></tr>
<tr><td>Between lab/opr variance, S_{Lj}^2</td><td colspan="4">0.7308</td><td>$S_{Lj}^2 = \dfrac{S_{dj}^2 - S_{rj}^2}{\bar{\bar{n}}_j}$</td></tr>
<tr><td>Reproducibility variance, S_{Rj}^2</td><td colspan="4">3.469</td><td>$S_{Rj}^2 = S_{rj}^2 + S_{Lj}^2$</td></tr>
<tr><td>S_{Rj}</td><td colspan="4">1.862</td><td></td></tr>
<tr><td>S_{rj}</td><td colspan="4">1.655</td><td></td></tr>
</table>

Repeatability = $S_r = \dfrac{1}{q}\sum_{j=1}^{q} S_{rj}$ **= 1.655MPa**

Reproducibility = $S_R = \frac{1}{q}\sum_{j=1}^{q} S_{Rj}$ **= 1.862MPa**

B1.7. Comments

Precision and Bias from Test Method

<u>SS 78: Part A16: 1987: Clause 7.2</u>

"For pairs of 150mm cubes made from the same sample, cured in similar conditions and tested at 28-days, the repeatability expressed as a percentage of the mean of the two cube strengths obtained, is 10% at the 95% probability level"

<u>BS 1881 : Part 116: 1983: Clause 7.3</u>

Table 1 Precision data for measurements of the compressive strength of hardened concrete, expressed as percentages of the mean of the two cube strengths whose differences is to be compared to r or R

Test Method	Repeatability Conditions		Reproducibility Conditions	
	s_r	r	s_R	R
	%	%	%	%
100 mm cubes	3.2	9.0	5.4	15.1
150 mm cubes	3.2	9.0	4.7	13.2

B1.8. Calculation of Measurement Uncertainty

Proficiency Test Data

From the proficiency test data, the repeatability standard deviation of the test method is estimated as 1.655 MPa and the reproducibility standard deviation of the test method is estimated as 1.862 MPa.

B1.8.1 Control of Bias

For a laboratory to show sufficient evidence of bias control, the standard deviation for proficiency testing has to be less than S_R and the laboratory has a mean z-score between $\pm\frac{2}{\sqrt{q}}$ for q assigned value.

Where excessive bias is detected, action will have to be taken to bring the bias within the required range before proceeding with measurements. Such action will involve investigation and elimination of the cause of the bias.

B1.8.2 Control of Precision

Laboratory has to demonstrate that its repeatability standard deviation is within the range found in the proficiency test. When this is the case, the precision is accordingly considered to be under good control.

B1.8.3 Measurement Uncertainty

The uncertainty $u(y)$ associated with an observation can be estimated using the following equation:

$$u^2(y) = u^2(\delta) + s_R^2 + \sum c_i^2 u_i^2(x_i)$$

where,

$$u(\delta) = s_\delta = \sqrt{\frac{s_R^2 - (1-1/n)s_r^2}{p}} = \sqrt{\frac{3.467-(1-1/6)2.739}{9}}$$

$$= 0.3628$$

$\sum c_i^2 u_i^2(x_i)$ = 0 (assuming that the controlling variables during the proficiency testing and routine testing remain constant)

Therefore,

$$u^2(y) = 0.3628^2 + 1.862^2 = 3.599$$

$$u(y) = 1.897$$

B1.8.4 Expanded Uncertainty

No. of test results per laboratory = 6

Total no. of test results in this proficiency study = 54

Degree of freedom, $v = 54 - 1 = 53$

From the Student's t table and for 95% Confidence Interval, the coverage factor k = 2.007 (from interpolation)

The expanded uncertainty, $U(y) = ku(y)$

$$= 2.007 \times 1.897$$

$$= 3.807 MPa$$

Therefore, the measurement uncertainty in the cube compressive strength is 3.81 MPa at 95% confidence level.

EXAMPLE (B2) - ULTRASONIC PULSE VELOCITY TEST ON WALL AND SLAB

B2.1. Introduction

This example serves to illustrate the estimation of measurement uncertainty according to ISO/TS 21748 approach. Repeatability and reproducibility information as required by the approach are obtained from proficiency testing data and calculated based on ISO 5725-2 (shown below from step B2.2 to step B2.6). The proficiency testing data is obtained from UPV Test on Wall and Slab tested in accordance to SS78: Part B3: 1992/BE EN 12504-4:2004.

Definition:

i	laboratory or operator	x	data set using mean
j	level	y	individual results
k	individual results by i	$\bar{y}$	mean of y of the laboratory or operator
p	no. of laboratory or operator	$\bar{\bar{y}}$	general mean of the level
q	no. of levels	s	spread of results, e.g. standard deviation
n	no. of results by each laboratory or operator		

B2.2. Original Data

Laboratory	Structural Element		
	Wall 1 (μs)	Slab 1 (μs)	…..j….q
Lab 1	36.4	37.2	
	36.6	36.8	
	36.5	36.9	
	36.3	37.5	
Lab 2	35.8	37.4	
	35.5	37.2	
	35.7	36.9	
	35.8	37.2	
Lab 3	36.5	38.5	
	35.4	38.7	
	35.0	37.1	
	35.8	37.6	
Lab 4	39.9	39.6	
	38.1	41.2	
	39.1	40.2	
	38.4	41.9	
Lab 5	36.2	36.8	
	35.7	36.4	
	36.0	37.8	
	37.6	36.6	
Lab 6	38.7	39.2	
	38.7	40.5	
	37.7	40.9	
	39.9	40.2	
Lab 7	39.6	40.8	
	40.1	40.4	

	38.4	41.4	
	37.9	40.2	
. . i . p			y_{ij1} , k=1 … y_{ijk} … $y_{ijn_{ij}}$, k=n_{ij}

B2.3. Mean for each Laboratory or Operator

Submission Lab or Operator	Structural Element				
	Wall 1		Slab 1		…..j….q
	$\overline{y}_{ij}$	n_{ij}	$\overline{y}_{ij}$	n_{ij}	
Lab 1	36.45	4	37.10	4	
Lab 2	35.70	4	37.18	4	
Lab 3	35.68	4	37.98	4	
Lab 4	38.88	4	40.73	4	
Lab 5	36.38	4	36.90	4	
Lab 6	38.75	4	40.20	4	
Lab 7	39.00	4	40.70	4	
. i . p					$\overline{y}_{ij} = \frac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} y_{ijk}$
General mean, $\hat{m}_j = \overline{\overline{y}}_j$	37.26		38.68		$\hat{m}_j = \overline{\overline{y}}_j = \frac{\sum_{i=1}^{p_j} \left(n_{ij} \times \overline{y_{ij}}\right)}{\sum_{i=1}^{p_j} n_{ij}}$

Note: Express to 1 more significant digit than original data

B2.4. Spread of each Cell (commonly sample standard deviation)

Submission Lab or Operator	Structural Element				
	Wall 1		Slab 1		…..j….q
	S_{ij}	S_{ij}^2	S_{ij}	S_{ij}^2	
Lab 1	0.1291	0.01667	0.3162	0.1000	
Lab 2	0.1414	0.02000	0.2062	0.04250	
Lab 3	0.6397	0.4092	0.7544	0.5692	
Lab 4	0.8016	0.6425	1.029	1.059	
Lab 5	0.8421	0.7092	0.6218	0.3867	
Lab 6	0.9000	0.8100	0.7257	0.5267	
Lab 7	1.023	1.047	0.5292	0.2800	
. i . p					$S_{ij} = \sqrt{\frac{1}{n_{ij}-1} \sum_{k=1}^{n_{ij}} \left(y_{ijk} - \overline{y}_{ij}\right)^2}$
$\sum_{i=1}^{p} S_{ij}^2$		3.654		2.964	

Note: Express to 1 more significant digit than original data

B2.5. Scrutiny of Results for Consistency and Outliers

- Graphical Method

Mandel's h statistics, for between-laboratory / between-operator consistency

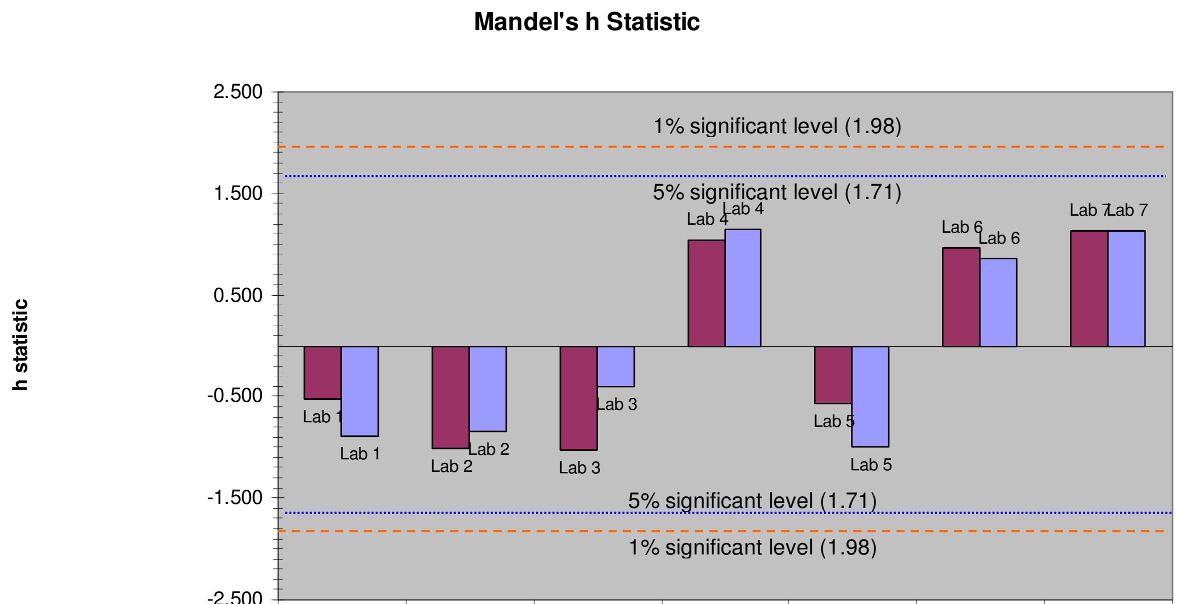
Submission Lab or Operator	Structural Element		
	Wall 1	**Slab 1**	…..j….q
Lab 1	-0.5262	-0.8890	
Lab 2	-1.013	-0.8469	
Lab 3	-1.029	-0.3977	
Lab 4	1.048	1.151	
Lab 5	-0.5749	-1.001	
Lab 6	0.9667	0.8517	
Lab 7	1.129	1.132	
. i . p			$h_{ij} = \frac{\overline{y_{ij}} - \overline{\overline{y_j}}}{\sqrt{\frac{1}{p_j - 1}\sum_{i=1}^{p_j}\left(\overline{y_{ij}} - \overline{\overline{y_j}}\right)^2}}$
p_j	7	7	n_j = no. of test results occurring in majority of the cells
n_j	4	4	
1% significance level	1.98		
5% significance level	1.71		

Mandel's k statistics, for within-laboratory / operator consistency

Submission Lab or Operator	Structural Element		
	Wall 1	**Slab 1**	…..j….q
Lab 1	0.1787	0.4860	
Lab 2	0.1957	0.3168	
Lab 3	0.8853	1.159	
Lab 4	1.109	1.581*	**Straggler**
Lab 5	1.166	0.9556	
Lab 6	1.246	1.115	
Lab 7	1.416	0.8132	
. i . p			$k_{ij} = \frac{S_{ij}\sqrt{p_j}}{\sqrt{\sum_{i=1}^{p_j} S_{ij}^2}}$
p_j	7	7	n_j = no. of test results occurring in majority of the cells
n_j	4	4	
1% significance level	1.79		
5% significance level	1.55		

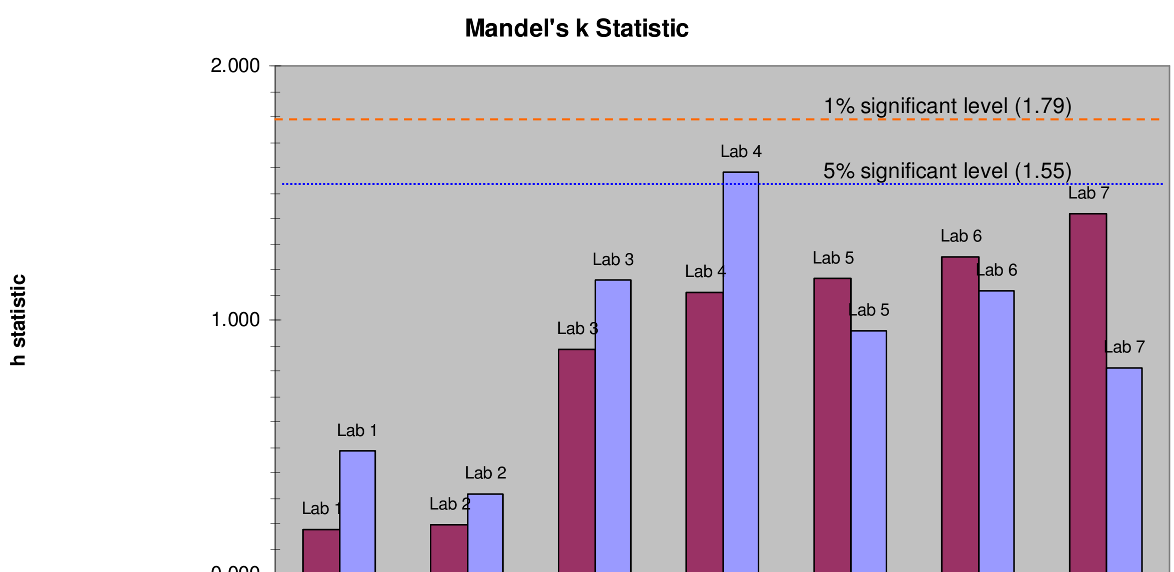
Mandel's h Test: All Laboratories results are less than 5% significance level.

Mandel's k Test: Straggler at Lab 4, Slab 1



	Lab 1	Lab 2	Lab 3	Lab 4	Lab 5	Lab 6	Lab 7
Mandel's h Level 1	-0.526	-1.013	-1.029	1.048	-0.575	0.967	1.129
Mandel's h Level 2	-0.889	-0.847	-0.398	1.151	-1.001	0.852	1.132

Laboratory



	Lab 1	Lab 2	Lab 3	Lab 4	Lab 5	Lab 6	Lab 7
Mandel's k Level 1	0.179	0.196	0.885	1.109	1.166	1.246	1.416
Mandel's k Level 2	0.486	0.317	1.159	1.581	0.956	1.115	0.813

Laboratory

- Numerical Method

Cochran’s test, for within laboratory / operator consistency. Test only the highest spread.

Submission Lab or Operator	Structural Element				
	Wall 1		Slab 1		…..j….q
	$S_{ij,\,max}$	C	$S_{ij,\,max}$	C	
Lab 1					
Lab 2					
Lab 3					
Lab 4			1.029	0.3573	**Acceptable**
Lab 5					
Lab 6					
Lab 7	1.023	0.2864			**Acceptable**
. i . p					$C = \dfrac{S^2_{ij,\,max}}{\sum_{i=1}^{pj} S_{ij}^2}$
p_j	7		7		
n_j	4		4		
Critical 1%	0.568		0.568		
Critical 5%	0.480		0.480		

Acceptable | * Stragglers | ** Outliers

C values | 5% | 1%

Cochran’s test: Lab 4, Slab 1 result is acceptable.
Lab 7, Wall 1 result is acceptable

Grubb’s Test

Grubb’s Test: Check single high,
Check single low,
If there is no outlier, then
Check double high,
Check double low.

Grubb’s test, for between-laboratory/between-operator consistency. Test single highest mean.

Single High Observation	Structural Element				
	Wall 1		Slab 1		…..j….q
	lab/opr	$\overline{y}_{ij}$	lab/opr	$\overline{y}_{ij}$	
x_1 =	Lab 3	35.68	Lab 5	36.90	$\overline{x} = \dfrac{1}{p_j}\sum_{i=1}^{pj} x_i$
…	Lab 2	35.70	Lab 1	37.10	
…	Lab 5	36.38	Lab 2	37.18	
…	Lab 1	36.45	Lab 3	37.98	
Sorted, x_i	Lab 6	38.75	Lab 6	40.20	
…	Lab 4	38.88	Lab 7	40.70	
x_p =	Lab 7	39.00	Lab 4	40.73	$S = \sqrt{\dfrac{1}{p_j - 1}\sum_{i=1}^{pj}\left(x_i - \overline{x}\right)^2}$
$\overline{x}$	37.26		38.68		
S	1.541		1.781		
G_p	1.129		1.151		
Critical 1%	2.139		2.139		$G_p = \dfrac{\left(x_p - \overline{x}\right)}{S}$
Critical 5%	2.020		2.020		

Acceptable | * Stragglers | ** Outliers

G values | 5% | 1%

Grubb's single high test: The highest mean is acceptable.

Grubb's test for between-laboratory / between-operator consistency. Test single lowest mean.

Single Low Observation	Structural Element				…..j….q
	Wall 1		Slab 1		
	lab/opr	$\overline{y}_{ij}$	lab/opr	$\overline{y}_{ij}$	
x_1=	Lab 3	35.68	Lab 5	36.90	$\overline{x} = \frac{1}{p_j}\sum_{i=1}^{p_j} x_i$
…	Lab 2	35.70	Lab 1	37.10	
…	Lab 5	36.38	Lab 2	37.18	
…	Lab 1	36.45	Lab 3	37.98	
Sorted, x_i	Lab 6	38.75	Lab 6	40.20	
…	Lab 4	38.88	Lab 7	40.70	$S = \sqrt{\frac{1}{p_j - 1}\sum_{i=1}^{p_j}\left(x_i - \overline{x}\right)^2}$
x_p=	Lab 7	39.00	Lab 4	40.73	
$\overline{x}$	37.26		38.68		
S	1.541		1.781		
G_1	1.029		1.001		
Critical 1%	2.139		2.139		$G_1 = \frac{\left(\overline{x} - x_1\right)}{S}$
Critical 5%	2.020		2.020		

Acceptable | * Stragglers | ** Outliers

G values | 5% | 1%

Grubb's single low test: All mean data acceptable.

Grubb's test, for between-laboratory / between-operator consistency. Test double highest mean.

Single High Observation	Structural Element				…..j….q
	Wall 1		Slab 1		
	lab/opr	$\overline{y}_{ij}$	lab/opr	$\overline{y}_{ij}$	
x_1 =	Lab 3	35.68	Lab 5	36.90	$\overline{x}_{p-1,p} = \frac{1}{p_j - 2}\sum_{i=1}^{p_j - 2} x_i$
…	Lab 2	35.70	Lab 1	37.10	
…	Lab 5	36.38	Lab 2	37.18	
…	Lab 1	36.45	Lab 3	37.98	
Sorted, x_i	Lab 6	38.75	Lab 6	40.20	$S^2_{p-1,p} = \sum_{i=1}^{p_j - 2}\left(x_i - \overline{x}_{p-1,p}\right)^2$
…	Lab 4		Lab 7		
x_p =	Lab 7		Lab 4		
$\overline{x}_{p-1,p}$	36.59		37.87		
$S^2_{p-1,p}$	6.361		7.457		$S^2_o = \sum_{i=1}^{p_j}\left(x_i - \overline{x}\right)^2$
S^2_o	14.24		19.03		
G_{2p}	0.4466		0.3918		
Critical 1%	0.0308		0.0308		$G_{2p} = \frac{S^2_{p-1,p}}{S^2_o}$
Critical 5%	0.0708		0.0708		

** Outliers | * Stragglers | Acceptable

G values | 1% | 5%

Grubb's double high test: All mean data acceptable.

Grubb's test, for between-laboratory / between-operator consistency. Test double lowest mean.

Single High Observation	Structural Element				
	Wall 1		Slab 1		…..j….q
	lab/opr	$\overline{y}_{ij}$	lab/opr	$\overline{y}_{ij}$	
x_1 =	Lab 3		Lab 5		$\overline{x}_{1,2} = \frac{1}{p_j - 2} \sum_{i=3}^{p_j} x_i$
…	Lab 2		Lab 1		
…	Lab 5	36.38	Lab 2	37.18	
…	Lab 1	36.45	Lab 3	37.98	
Sorted, x_i	Lab 6	38.75	Lab 6	40.20	$S^2_{1,2} = \sum_{i=3}^{p_j} \left(x_i - \overline{x}_{1,2}\right)^2$
…	Lab 4	38.88	Lab 7	40.70	
x_p =	Lab 7	39.00	Lab 4	40.73	
$\overline{x}_{1,2}$	37.89		39.36		
$S^2_{1,2}$	7.311		11.08		$S^2_o = \sum_{i=1}^{p_j} \left(x_i - \overline{x}\right)^2$
S^2_o	14.24		19.03		
$G_{1,2}$	0.5134		0.5821		
Critical 1%	0.0308		0.0308		$G_{1,2} = \frac{S^2_{1,2}}{S^2_o}$
Critical 5%	0.0708		0.0708		

** Outliers | * Stragglers | Acceptable

G values | 1% | 5%

Grubb's double low test: All mean data acceptable.

B2.6. Calculation of Mean, Repeatability and Reproducibility

Submission Lab or Operator	Structural Element								
	Wall 1				Slab 1				…..j….q
	$\bar{y}_{ij}$	S_{ij}	S_{ij}^2	n_{ij}	$\bar{y}_{ij}$	S_{ij}	S_{ij}^2	n_{ij}	
Lab 1	36.45	0.1291	0.01667	4	37.10	0.3162	0.1000	4	
Lab 2	35.70	0.1414	0.02000	4	37.18	0.2061	0.04250	4	
Lab 3	35.68	0.6397	0.4092	4	37.98	0.7544	0.5692	4	
Lab 4	38.88	0.8016	0.6425	4	40.73	1.029	1.059	4	
Lab 5	36.38	0.8421	0.7092	4	36.90	0.6218	0.3867	4	
Lab 6	38.75	0.9000	0.8100	4	40.20	0.7257	0.5267	4	
Lab 7	39.00	1.023	1.047	4	40.70	0.5292	0.2800	4	
. i . p									

	Wall 1	Slab 1	
p_j	7	7	
General mean, $\hat{m}_j = \bar{\bar{y}}_j$	37.26	38.68	$\hat{m}_j = \bar{\bar{y}}_j = \dfrac{\sum_{i=1}^{p_j}\left(n_{ij}\times\bar{y}_{ij}\right)}{\sum_{i=1}^{p_j} n_{ij}}$
Repeatability variance, S_{rj}^2	0.5220	0.4234	$S_{rj}^2 = \dfrac{\sum_{i=1}^{p_j}\left[(n_{ij}-1)xS_{ij}^2\right]}{\sum_{i=1}^{p_j}(n_{ij}-1)}$
$\bar{\bar{n}}_j$	4.0	4.0	$\bar{\bar{n}}_j = \dfrac{1}{p_j-1}\left(\sum_{i=1}^{p_j} n_{ij} - \dfrac{\sum_{i=1}^{p_j} n_{ij}^2}{\sum_{i=1}^{p_j} n_{ij}}\right)$
S_{dj}^2	9.494	12.69	$S_{dj}^2 = \dfrac{1}{p_j-1}\left[\sum_{i=1}^{p_j}\left(n_{ij}\bar{y}_{ij}^2\right) - \bar{\bar{y}}_j^2 \sum_{i=1}^{p_j} n_{ij}\right]$
Between lab/opr variance, S_{Lj}^2	2.243	3.066	$S_{Lj}^2 = \dfrac{S_{dj}^2 - S_{rj}^2}{\bar{\bar{n}}_j}$
Reproducibility variance, S_{Rj}^2	2.765	3.489	$S_{Rj}^2 = S_{rj}^2 + S_{Lj}^2$
S_{Rj}	1.663	1.868	
S_{rj}	0.7225	0.6507	

B2.7. Calculation of Measurement Uncertainty

B2.7.1 Proficiency Test Data

From the proficiency test data, the repeatability standard deviation of the test method is estimated as 0.7225 μs (Wall 1) and 0.6507 μs (Slab 1) and the reproducibility standard deviation of the test method is estimated as 1.663 μs (Wall 1) and 1.868 μs (Slab 1).

B2.7.2 Control of Bias

For a laboratory to show sufficient evidence of bias control, the standard deviation for proficiency testing has to be less than S_R and the laboratory has a mean z-score between $\pm\frac{2}{\sqrt{q}}$ for q assigned value

Where excessive bias is detected, action will have to be taken to bring the bias within the required range before proceeding with measurements. Such action will involve investigation and elimination of the cause of the bias.

B2.7.3 Control of Precision

Laboratory has to demonstrate that its repeatability standard deviation is within the range found in the proficiency test. When this is the case, the precision is accordingly considered to be under good control.

B2.7.4 Measurement Uncertainty

The uncertainty $u(y)$ associated with an observation can be estimated using the following equation:

$$u^2(y) = u^2(\delta) + s_R^2 + \sum c_i^2 u_i^2(x_i)$$

where,

$$u(\delta) = s_\delta = \sqrt{\frac{s_R^2 - (1-\frac{1}{n})s_r^2}{p}}$$

$\sum c_i^2 u_i^2(x_i)$ = 0 (assuming that the controlling variables during the proficiency testing and routine testing remain constant)

For Wall 1

$$u(\delta) = s_\delta = \sqrt{\frac{s_R^2 - (1-\frac{1}{n})s_r^2}{p}} = \sqrt{\frac{2.765 - (1-\frac{1}{4})0.5220}{7}} = 0.5824$$

Therefore,

$$u^2(y) = 0.5824^2 + 1.663^2 = 3.105$$

$$u(y) = 1.762$$

For Slab 1

$$u(\delta) = s_\delta = \sqrt{\frac{s_R^2 - (1-\frac{1}{n})s_r^2}{p}} = \sqrt{\frac{3.489 - (1-\frac{1}{4})0.4234}{7}} = 0.6731$$

Therefore,

$$u^2(y) = 0.6731^2 + 1.868^2 = 3.942$$

$$u(y) = 1.986$$

B2.7.5 Expanded Uncertainty

No. of test results per laboratory = 4

Total no. of test results per structural element in this proficiency study = 28

Degree of freedom, $v = 28 - 1 = 27$

From the Student's t table and for 95% Confidence Interval, the coverage factor k = 2.052 (from interpolation)

For Wall 1

The expanded uncertainty, $U(y) = ku(y)$

$$= 2.052 \times 1.762$$

$$= 3.616 \ \mu\text{s}$$

For Slab 1

The expanded uncertainty, $U(y) = ku(y)$

$$= 2.052 \times 1.986$$

$$= 4.075 \ \mu\text{s}$$

Therefore, the measurement uncertainty in the UPV measurement on Wall 1 and Slab 1 is 3.62 µs and 4.08 µs respectively at 95% confidence level.

EXAMPLE (B3) - WINDSOR PROBE TEST ON WALL

B3.1. Introduction

This example serves to illustrate the estimation of measurement uncertainty according to ISO/TS 21748 approach. Repeatability and reproducibility information as required by the approach are obtained from proficiency testing data and calculated based on ISO 5725-2 (shown below from step B3.2 to step B3.6). The proficiency testing data is obtained from Windsor Probe tested in accordance to BS1881: Part 207:1992/ASTM C803/C803M-03.

Definition:

i	laboratory or operator	x	data set using mean
j	level	y	individual results
k	individual results by i	$\bar{y}$	mean of y of the laboratory or operator
p	no. of laboratory or operator	$\bar{\bar{y}}$	general mean of the level
q	no. of levels	s	spread of results, e.g. standard deviation
n	no. of results by each laboratory or operator		

B3.2. Original Data

Laboratory	Level	
	Wall 2 Face B (mm)	…..j….q
Lab 1	-	
	-	
	-	
	-	
Lab 2	49.2	
	47.7	
	49.6	
	51.8	
Lab 3	56.1	
	57.4	
	58.1	
	54.6	
Lab 4	51.5	
	53.0	
	52.2	
	50.7	
Lab 5	52.5	
	54.0	
	52.0	
	53.0	
Lab 6	49.4	
	48.5	
	50.3	
	52.5	
Lab 7	46.5	
	51.5	
	48.4	
	49.5	

. . i . p		y_{ij1}, k=1 … y_{ijk} … $y_{ijn_{ij}}$, $k=n_{ij}$

B3.3. Mean for each Laboratory or Operator

Submission Lab or Operator	Level		
	Wall 2 Face B		…..j….q
	$\overline{y_{ij}}$	n_{ij}	
Lab 1	-	-	
Lab 2	49.58	4	
Lab 3	56.55	4	
Lab 4	51.85	4	
Lab 5	52.88	4	
Lab 6	50.18	4	
Lab 7	48.98	4	
. i . p			$\overline{y_{ij}} = \frac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} y_{ijk}$
General mean, $\widehat{m}_j = \overline{\overline{y_j}}$	51.67		$\widehat{m}_j = \overline{\overline{y_j}} = \frac{\sum_{i=1}^{p_j} \left(n_{ij} \times \overline{y_{ij}}\right)}{\sum_{i=1}^{p_j} n_{ij}}$

Note: Express to 1 more significant digit than original data

B3.4. Spread of each Cell (commonly sample standard deviation)

Submission Lab or Operator	Level		
	Wall 2 Face B		…..j….q
	S_{ij}	S_{ij}^2	
Lab 1	-	-	
Lab 2	1.694	2.869	
Lab 3	1.542	2.377	
Lab 4	0.9815	0.9633	
Lab 5	0.8539	0.7292	
Lab 6	1.715	2.942	
Lab 7	2.090	4.369	
. i . p			$S_{ij} = \sqrt{\frac{1}{n_{ij}-1} \sum_{k=1}^{n_{ij}} \left(y_{ijk} - \overline{y_{ij}}\right)^2}$
$\sum_{i=1}^{p} S_{ij}^2$		14.25	

Note: Express to 1 more significant digit than original data

B3.5. Scrutiny of Results for Consistency and Outliers

- Graphical Method

Mandel's h statistics, for between-laboratory / between-operator consistency

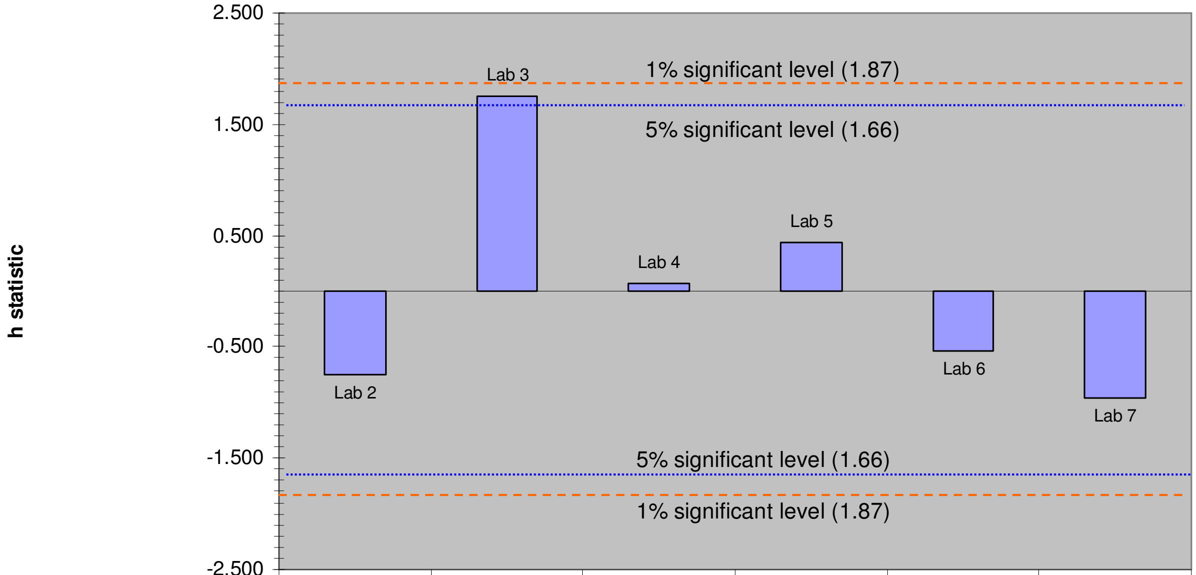
Submission Lab or Operator	Level	
	Wall 2 Face B	…..j….q
Lab 1	-	
Lab 2	-0.7472	
Lab 3	1.744*	**Straggler**
Lab 4	0.06549	
Lab 5	0.4316	
Lab 6	-0.5328	
Lab 7	-0.9615	
. i . p		$h_{ij} = \dfrac{\overline{y_{ij}} - \overline{\overline{y_j}}}{\sqrt{\dfrac{1}{p_j - 1}\sum_{i=1}^{p_j}\left(\overline{y_{ij}} - \overline{\overline{y_j}}\right)^2}}$
p_j	6	n_j = no. of test results occurring in majority of the cells
n_j	4	
1% significance level	1.87	
5% significance level	1.66	

Mandel's k statistics, for within-laboratory / operator consistency

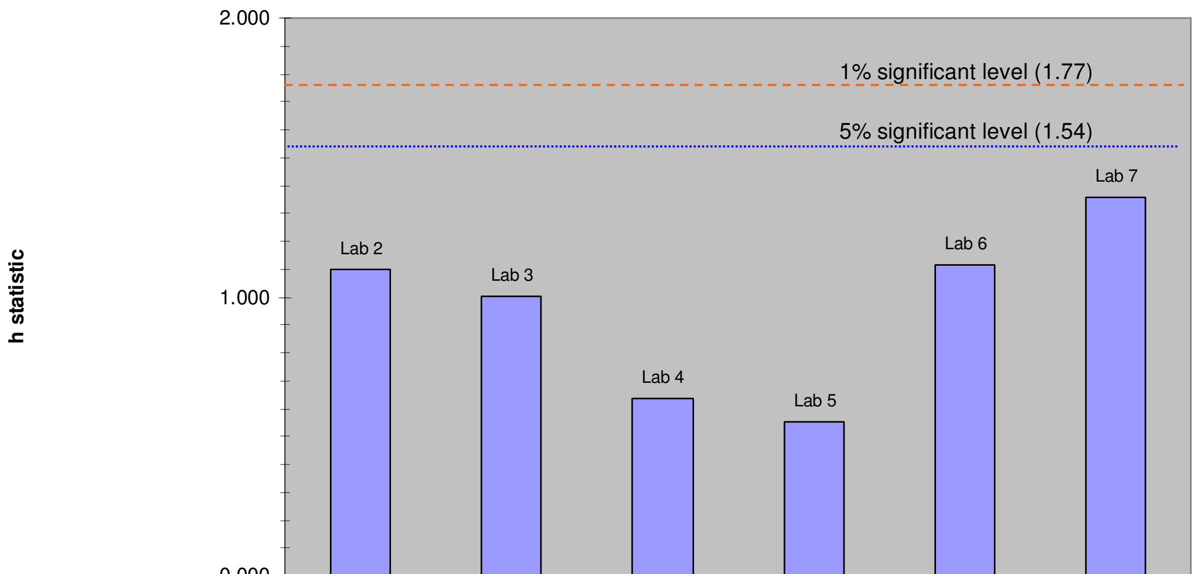
Submission Lab or Operator	Level	
	Wall 2 Face B	…..j….q
Lab 1	-	
Lab 2	1.099	
Lab 3	1.000	
Lab 4	0.6369	
Lab 5	0.5541	
Lab 6	1.113	
Lab 7	1.356	
. i . p		$k_{ij} = \dfrac{S_{ij}\sqrt{p_j}}{\sqrt{\sum_{i=1}^{p_j} S_{ij}^2}}$
p_j	6	n_j = no. of test results occurring in majority of the cells
n_j	4	
1% significance level	1.77	
5% significance level	1.54	

Mandel's h Test: Straggler at Lab 3.

Mandel's k Test: All laboratories results are less than 5% significance level.



	Lab 2	Lab 3	Lab 4	Lab 5	Lab 6	Lab 7
Mandel's h Level 1	-0.747	1.744	0.065	0.432	-0.533	-0.961

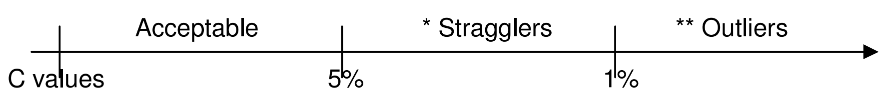


	Lab 2	Lab 3	Lab 4	Lab 5	Lab 6	Lab 7
Mandel's k Level 1	1.099	1.000	0.637	0.554	1.113	1.356

- Numerical Method

Cochran's test, for within laboratory / operator consistency. Test only the highest spread.

Submission Lab or Operator	Level		
	Wall 2 Face B		…..j….q
	$S_{ij,\,max}$	C	
Lab 1			
Lab 2			
Lab 3			
Lab 4			
Lab 5			
Lab 6			
Lab 7	2.090	0.3066	**Acceptable**
. i . p			$C = \dfrac{S^2_{ij,\,max}}{\sum_{i=1}^{p_j} S_{ij}^2}$
p_j	6		
n_j	4		
Critical 1%	0.626		
Critical 5%	0.532		



Cochran's test: Lab 7 result is acceptable

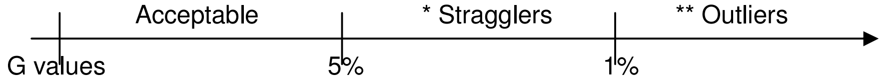
Grubb's Test

Grubb's Test: Check single high
Check single low
If there is no outlier, then
Check double high
Check double low

Grubb's test, for between-laboratory/between-operator consistency. Test single highest mean.

Single High Observation	Level		
	Wall 2 Face B		…..j….q
	lab/opr	$\overline{y}_{ij}$	
x_1 =	Lab 7	48.98	$\overline{x} = \dfrac{1}{p_j}\sum_{i=1}^{p_j} x_i$
…	Lab 2	49.58	
…	Lab 6	50.18	
Sorted, x_i	Lab 4	51.85	
…	Lab 5	52.88	
x_p =	<u>Lab 3</u>	<u>56.55</u>	$S = \sqrt{\dfrac{1}{p_j - 1}\sum_{i=1}^{p_j}\left(x_i - \overline{x}\right)^2}$
$\overline{x}$	51.67		
S	2.800		
G_p	1.744		
Critical 1%	1.973		

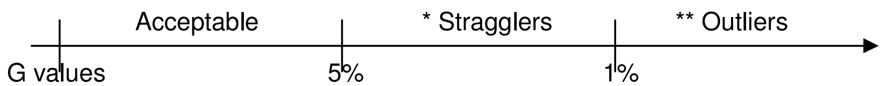
Critical 5%	1.887	$G_p = \frac{(x_p - \bar{x})}{S}$



Grubb's single high test: The highest mean is acceptable.

Grubb's test for between-laboratory / between-operator consistency. Test single lowest mean.

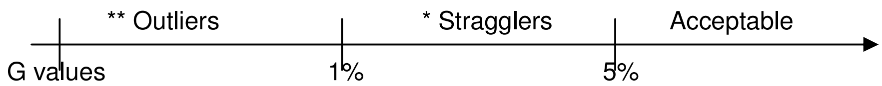
Single Low Observation	Level		
	Wall 2 Face B		…..j….q
	lab/opr	$\bar{y}_{ij}$	
x_1=	Lab 7	48.98	$\bar{x} = \frac{1}{p_j}\sum_{i=1}^{p_j} x_i$
…	Lab 2	49.58	
Sorted, x_i	Lab 6	50.18	
…	Lab 4	51.85	
…	Lab 5	52.88	
x_p=	Lab 3	56.55	
$\bar{x}$	51.67		$S = \sqrt{\frac{1}{p_j - 1}\sum_{i=1}^{p_j}(x_i - \bar{x})^2}$
S	2.800		
G_1	0.9615		
Critical 1%	1.973		
Critical 5%	1.887		$G_1 = \frac{(\bar{x} - x_1)}{S}$



Grubb's single low test: All mean data acceptable.

Grubb's test, for between-laboratory / between-operator consistency. Test double highest mean.

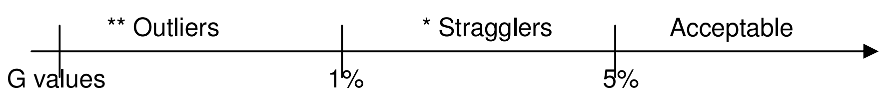
Single High Observation	Level		
	Wall 2 Face B		…..j….q
	lab/opr	$\bar{y}_{ij}$	
x_1 =	Lab 7	48.98	$\bar{x}_{p-1,p} = \frac{1}{p_j - 2}\sum_{i=1}^{p_j - 2} x_i$
…	Lab 2	49.58	
Sorted, x_i	Lab 6	50.18	
…	Lab 4	51.85	$S^2_{p-1,p} = \sum_{i=1}^{p_j - 2}(x_i - \bar{x}_{p-1,p})^2$
…	Lab 5		
x_p =	Lab 3		
$\bar{x}_{p-1,p}$	50.14		
$S^2_{p-1,p}$	4.602		$S^2_o = \sum_{i=1}^{p_j}(x_i - \bar{x})^2$
S^2_o	39.18		
G_{2p}	0.1174		
Critical 1%	0.0116		$G_{2p} = \frac{S^2_{p-1,p}}{S^2_o}$
Critical 5%	0.0349		



Grubb's double high test: All mean data acceptable.

Grubb's test, for between-laboratory / between-operator consistency. Test double lowest mean.

Single High Observation	Level		
	Wall 2 Face B		…..j….q
	lab/opr	$\bar{y}_{ij}$	
x_1 =	Lab 7		$\bar{x}_{1,2} = \frac{1}{p_j - 2}\sum_{i=3}^{p_j} x_i$
…	Lab 2		
…	Lab 6	50.18	
Sorted, x_i	Lab 4	51.85	$S^2_{1,2} = \sum_{i=3}^{p_j}\left(x_i - \bar{x}_{1,2}\right)^2$
…	Lab 5	52.88	
x_p =	Lab 3	56.55	
$\bar{x}_{1,2}$	52.86		
$S^2_{1,2}$	21.84		$S^2_o = \sum_{i=1}^{p_j}\left(x_i - \bar{x}\right)^2$
S^2_o	39.18		
$G_{1,2}$	0.5575		
Critical 1%	0.0116		$G_{1,2} = \frac{S^2_{1,2}}{S^2_o}$
Critical 5%	0.0349		



Grubb's double low test: All mean data acceptable.

B3.6. Calculation of Mean, Repeatability and Reproducibility

Submission Lab or Operator	**Level**				
	Wall 2 Face B				…..j….q
	$\bar{y}_{ij}$	S_{ij}	S_{ij}^2	n_{ij}	
Lab 2	49.58	1.694	2.869	4	
Lab 3	56.55	1.542	2.377	4	
Lab 4	51.85	0.9815	0.9633	4	
Lab 5	52.88	0.8539	0.7292	4	
Lab 6	50.18	1.715	2.9425	4	
Lab 7	48.98	2.090	4.3692	4	
. i . p					
p_j	6				
General mean, $\hat{m}_j = \bar{\bar{y}}_j$	51.67				$\hat{m}_j = \bar{\bar{y}}_j = \dfrac{\sum_{i=1}^{p_j}\left(n_{ij} \times \bar{y}_{ij}\right)}{\sum_{i=1}^{p_j} n_{ij}}$
Repeatability variance, S_{rj}^2	2.375				$S_{rj}^2 = \dfrac{\sum_{i=1}^{p_j}\left[\left(n_{ij}-1\right) x S_{ij}^2\right]}{\sum_{i=1}^{p_j}\left(n_{ij}-1\right)}$
$\bar{\bar{n}}_j$	4.0				$\bar{\bar{n}}_j = \dfrac{1}{p_j-1}\left(\sum_{i=1}^{p_j} n_{ij} - \dfrac{\sum_{i=1}^{p_j} n_{ij}^2}{\sum_{i=1}^{p_j} n_{ij}}\right)$
S_{dj}^2	31.35				$S_{dj}^2 = \dfrac{1}{p_j-1}\left[\sum_{i=1}^{p_j}\left(n_{ij}\bar{y}_{ij}^2\right) - \bar{\bar{y}}_j^2 \sum_{i=1}^{p_j} n_{ij}\right]$
Between lab/opr variance, S_{Lj}^2	7.243				$S_{Lj}^2 = \dfrac{S_{dj}^2 - S_{rj}^2}{\bar{\bar{n}}_j}$
Reproducibility variance, S_{Rj}^2	9.618				$S_{Rj}^2 = S_{rj}^2 + S_{Lj}^2$
S_{Rj}	3.101				
S_{rj}	1.541				

B3.7. Calculation of Measurement Uncertainty

B3.7.1 Proficiency Test Data

From the proficiency test data, the repeatability standard deviation of the test method is estimated as 1.541 mm and the reproducibility standard deviation of the test method is estimated as 3.101 mm.

B3.7.2 Control of Bias

For a laboratory to show sufficient evidence of bias control, the standard deviation for proficiency testing has to be less than S_R and the laboratory has a mean z-score between $\pm\frac{2}{\sqrt{q}}$ for q assigned value

Where excessive bias is detected, action will have to be taken to bring the bias within the required range before proceeding with measurements. Such action will involve investigation and elimination of the cause of the bias.

B3.7.3 Control of Precision

Laboratory has to demonstrate that its repeatability standard deviation is within the range found in the proficiency test. When this is the case, the precision is accordingly considered to be under good control.

B3.7.4 Measurement Uncertainty

The uncertainty $u(y)$ associated with an observation can be estimated using the following equation:

$$u^2(y) = u^2(\delta) + s_R^2 + \sum c_i^2 u_i^2(x_i)$$

where,

$$u(\delta) = s_\delta = \sqrt{\frac{s_R^2 - (1-\frac{1}{n})s_r^2}{p}}$$

$\sum c_i^2 u_i^2(x_i)$ = 0 (assuming that the controlling variables during the proficiency testing and routine testing remain constant)

Thus,

$$u(\delta) = s_\delta = \sqrt{\frac{s_R^2 - (1-\frac{1}{n})s_r^2}{p}} = \sqrt{\frac{9.616 - (1-\frac{1}{4})2.375}{6}} = 1.143$$

Therefore,

$$u^2(y) = 1.143^2 + 3.101^2 = 10.92$$

$$u(y) = 3.305$$

B3.7.5 Expanded Uncertainty

No. of test results per laboratory = 4

Total no. of test results in this proficiency study = 24

Degree of freedom, $v = 24 - 1 = 23$

From the Student's t table and for 95% Confidence Interval, the coverage factor k = 2.072 (from interpolation)

The expanded uncertainty, $U(y) = ku(y)$

$$= 2.072 \times 3.305$$

$$= 6.848 \text{ mm}$$

Therefore, the measurement uncertainty in the WP measurement is 6.85 mm at 95% confidence level.

EXAMPLE (B4) - REBOUND HAMMER TEST ON WALL AND SLAB

B4.1. Introduction

This example serves to illustrate the estimation of measurement uncertainty according to ISO/TS 21748 approach. Repeatability and reproducibility information as required by the approach are obtained from proficiency testing data and calculated based on ISO 5725-2 (shown below from step B4.2 to step B4.6). The proficiency testing data is obtained from Rebound Hammer Test on Wall and Slab tested in accordance to SS78: Part B2: 1992 / BS EN 12504-2:2001.

Definition:

i	laboratory or operator	x	data set using mean
j	level, e.g. targeted structural elements	y	individual results
k	individual results by i	$\bar{y}$	mean of y of the laboratory or operator
p	no. of laboratory or operator	$\bar{\bar{y}}$	general mean of the level
q	no. of levels	s	spread of results, e.g. standard deviation
n	no. of results by each laboratory or operator		

B4.2. Original Data

Laboratory	Structural Element		
	Wall 1	Slab 1	…..j….q
Lab 1	39.0	41.0	
	38.5	40.5	
	38.5	40.5	
	39.5	41.0	
Lab 2	39.0	39.0	
	37.7	39.5	
	37.9	39.0	
	38.6	39.5	
Lab 3	48.0	49.0	
	48.0	50.0	
	47.0	50.0	
	48.0	49.0	
Lab 4	34.0	39.5	
	34.0	36.0	
	35.0	37.0	
	34.0	35.0	
Lab 5	39.0	43.5	
	38.0	45.0	
	39.0	42.5	
	39.0	42.5	
Lab 6	31.0	34.0	
	32.0	36.0	
	33.0	36.0	
	33.0	34.0	
Lab 7	31.0	35.0	
	32.0	35.0	
	33.0	36.0	
	31.0	35.0	

. . i . p			y_{ij1} , k=1 … y_{ijk} … $y_{ijn_{ij}}$, $k=n_{ij}$

B4.3. Mean for each Laboratory or Operator

Submission Lab or Operator	Structural Element				
	Wall 1		Slab 1		…..j….q
	$\bar{y}_{ij}$	n_{ij}	$\bar{y}_{ij}$	n_{ij}	
Lab 1	38.88	4	40.75	4	
Lab 2	38.30	4	39.25	4	
Lab 3	47.75	4	49.50	4	
Lab 4	34.25	4	36.88	4	
Lab 5	38.75	4	43.38	4	
Lab 6	32.25	4	35.00	4	
Lab 7	31.75	4	35.25	4	
. i . p					$\bar{y}_{ij} = \frac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} y_{ijk}$
General mean, $\hat{m}_j = \bar{\bar{y}}_j$	37.42		40.00		$\hat{m}_j = \bar{\bar{y}}_j = \frac{\sum_{i=1}^{p_j} \left(n_{ij} \times \bar{y}_{ij}\right)}{\sum_{i=1}^{p_j} n_{ij}}$

Note: Express to 1 more significant digit than original data

B4.4. Spread of each Cell (commonly sample standard deviation)

Submission Lab or Operator	Structural Element				
	Wall 1		Slab 1		…..j….q
	S_{ij}	S_{ij}^2	S_{ij}	S_{ij}^2	
Lab 1	0.4787	0.2292	0.2887	0.08333	
Lab 2	0.6055	0.3667	0.2887	0.08333	
Lab 3	0.5000	0.2500	0.5774	0.3333	
Lab 4	0.5000	0.2500	1.931	3.729	
Lab 5	0.5000	0.2500	1.181	1.396	
Lab 6	0.9574	0.9167	1.155	1.333	
Lab 7	0.9574	0.9167	0.5000	0.2500	
. i . p					$S_{ij} = \sqrt{\frac{1}{n_{ij}-1} \sum_{k=1}^{n_{ij}} \left(y_{ijk} - \bar{y}_{ij}\right)^2}$
$\sum_{i=1}^{p} S_{ij}^2$		3.179		7.208	

Note: Express to 1 more significant digit than original data

B4.5. Scrutiny of Results for Consistency and Outliers

- Graphical Method

Mandel's h statistics, for between-laboratory / between-operator consistency

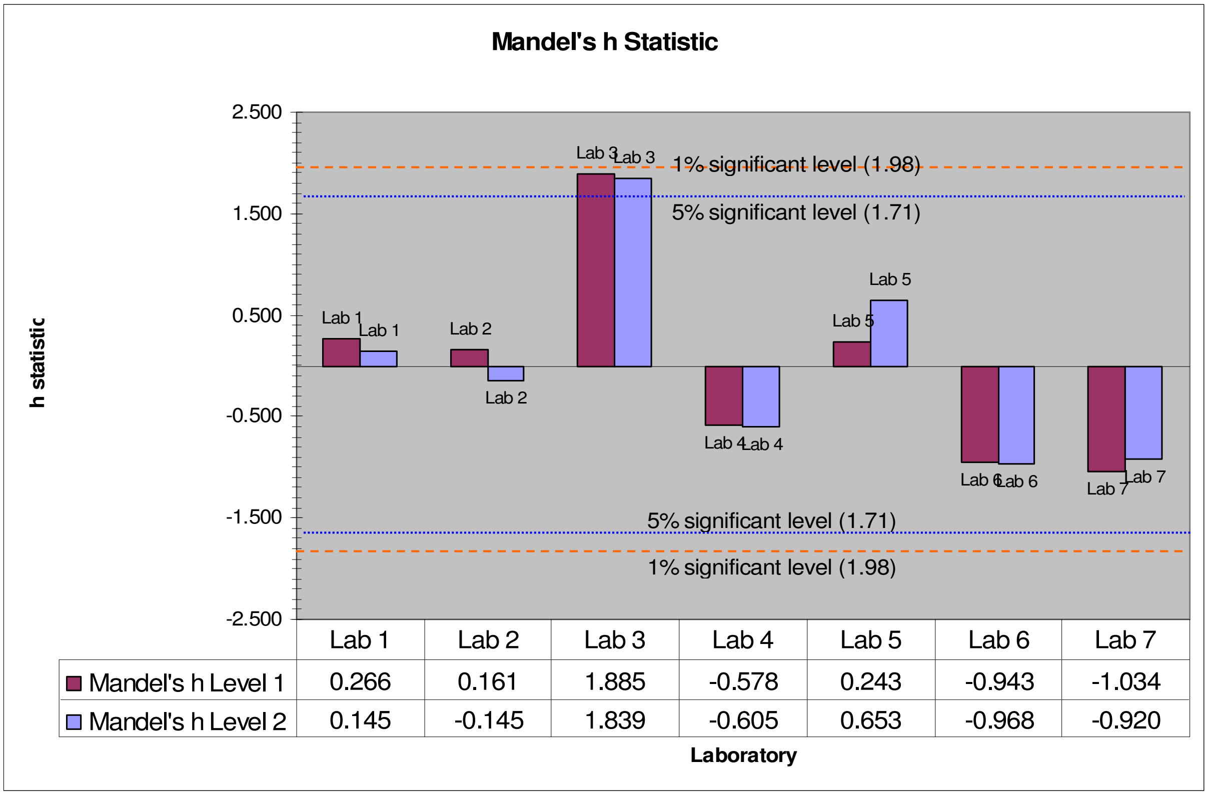
Submission Lab or Operator	Structural Element		
	Wall 1	**Slab 1**	…..j….q
Lab 1	0.2658	0.1452	
Lab 2	0.1609	-0.1452	
Lab 3	1.885*	1.839*	**Straggler**
Lab 4	-0.5779	-0.6050	
Lab 5	0.2430	0.6534	
Lab 6	-0.9427	-0.9680	
Lab 7	-1.034	-0.9196	
. i . p			$h_{ij} = \dfrac{\overline{y_{ij}} - \overline{\overline{y_j}}}{\sqrt{\dfrac{1}{p_j - 1}\sum_{i=1}^{p_j}\left(\overline{y_{ij}} - \overline{\overline{y_j}}\right)^2}}$
p_j	7	7	n_j = no. of test results occurring in majority of the cells
1% significance level	1.98	1.98	
5% significance level	1.71	1.71	

Mandel's k statistics, for within-laboratory / operator consistency

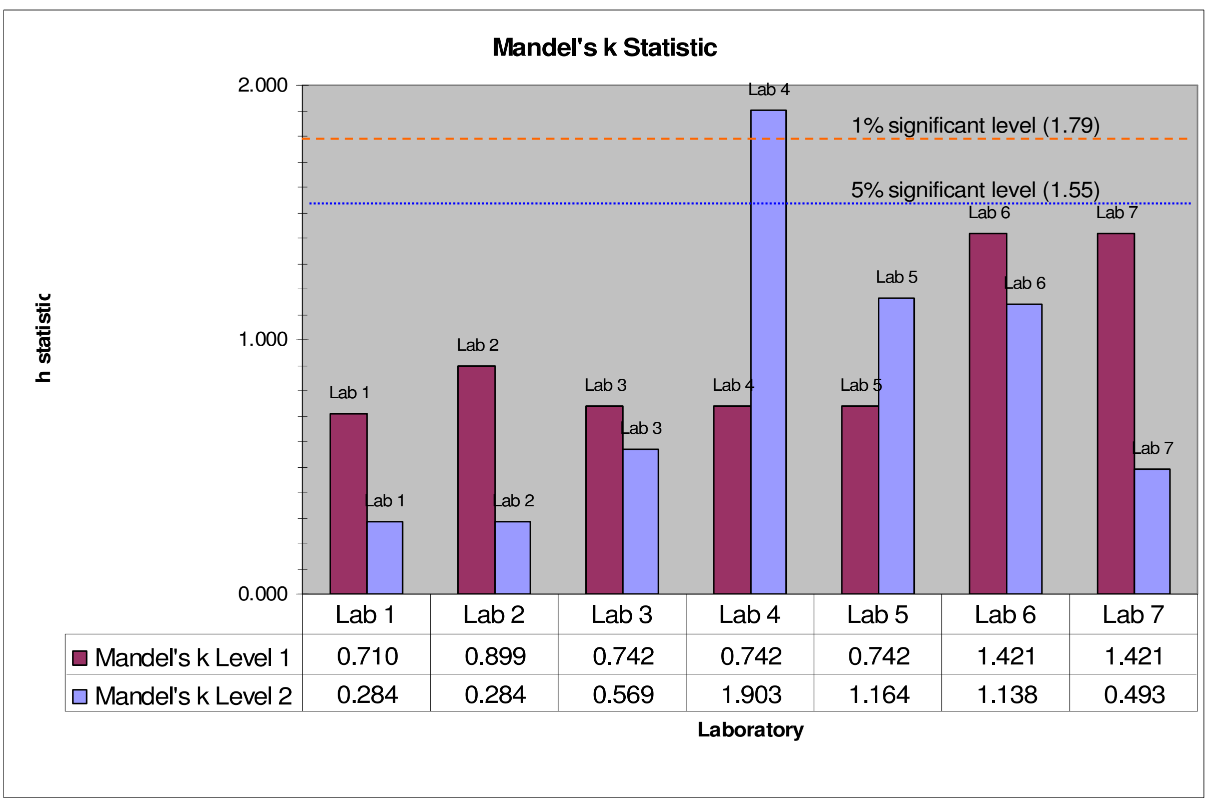
Submission Lab or Operator	Structural Element		
	Wall 1	**Slab 1**	…..j….q
Lab 1	0.7103	0.2845	
Lab 2	0.8985	0.2845	
Lab 3	0.7419	0.5690	
Lab 4	0.7419	1.903**	**Outlier**
Lab 5	0.7419	1.164	
Lab 6	1.421	1.138	
Lab 7	1.421	0.4927	
. i . p			$k_{ij} = \dfrac{S_{ij}\sqrt{p_j}}{\sqrt{\sum_{i=1}^{p_j} S_{ij}^2}}$
p_j	7	7	n_j = no. of test results occurring in majority of the cells
n_j	4	4	
1% significance level	1.79	1.79	
5% significance level	1.55	1.55	

Mandel's h Test: Straggler at Lab 3 for both Wall 1 and Slab 1.

Mandel's k Test: Outlier at Lab 4, Slab 1.



	Lab 1	Lab 2	Lab 3	Lab 4	Lab 5	Lab 6	Lab 7
Mandel's h Level 1	0.266	0.161	1.885	-0.578	0.243	-0.943	-1.034
Mandel's h Level 2	0.145	-0.145	1.839	-0.605	0.653	-0.968	-0.920



	Lab 1	Lab 2	Lab 3	Lab 4	Lab 5	Lab 6	Lab 7
Mandel's k Level 1	0.710	0.899	0.742	0.742	0.742	1.421	1.421
Mandel's k Level 2	0.284	0.284	0.569	1.903	1.164	1.138	0.493

- Numerical Method

Cochran's test, for within laboratory / operator consistency. Test only the highest spread.

Submission Lab or Operator	Structural Element				
	Wall 1		Slab 1		…..j….q
	$S_{ij, max}$	C	$S_{ij, max}$	C	
Lab 1					
Lab 2					
Lab 3					
Lab 4			1.931	0.5173	**Acceptable**
Lab 5					
Lab 6					
Lab 7	0.9574	0.2883			**Acceptable**
. i . p					$C = \frac{S^2_{ij, max}}{\sum_{i=1}^{p_j} S_{ij}^2}$
p_j	7		7		
n_j	4		4		
Critical 1%	0.568		0.568		
Critical 5%	0.480		0.480		

Cochran's test: Lab 4, Slab 1 result is a straggler.
Lab 7, Wall 1 result is acceptable

Grubb's Test

Grubb's Test: Check single high,
Check single low,
If there is no outlier, then
Check double high
Check double low.

Grubb's test, for between-laboratory/between-operator consistency. Test single highest mean.

Single High Observation	Structural Element				
	Wall 1		Slab 1		…..j….q
	lab/opr	$\overline{y_{ij}}$	lab/opr	$\overline{y_{ij}}$	
x_1 =	Lab 7	31.75	Lab 6	35.00	$\bar{x} = \frac{1}{p_j}\sum_{i=1}^{p_j} x_i$
…	Lab 6	32.25	Lab 7	35.25	
…	Lab 4	34.25	Lab 4	36.88	
…	Lab 2	38.30	Lab 2	39.25	
Sorted, x_i	Lab 5	38.75	Lab 1	40.75	
…	Lab 1	38.88	Lab 5	43.38	
x_p =	Lab 3	47.75	Lab 3	49.50	$S = \sqrt{\frac{1}{p_j - 1}\sum_{i=1}^{p_j}(x_i - \bar{x})^2}$
$\bar{x}$	37.42		40.00		
S	5.482		5.165		
G_p	1.885		1.839		

Critical 1%	2.139	2.139	$G_p = \frac{(x_p - \bar{x})}{S}$
Critical 5%	2.020	2.020	

Acceptable * Stragglers ** Outliers

G values 5% 1%

Grubb's single high test: The highest mean is acceptable.

Grubb's test for between-laboratory / between-operator consistency. Test single lowest mean.

Single Low Observation	Structural Element				
	Wall 1		Slab 1		…..j….q
	lab/opr	$\bar{y}_{ij}$	lab/opr	$\bar{y}_{ij}$	
x_1=	Lab 7	31.75	Lab 6	35.00	$\bar{x} = \frac{1}{p_j}\sum_{i=1}^{p_j} x_i$
…	Lab 6	32.25	Lab 7	35.25	
…	Lab 4	34.25	Lab 4	36.88	
…	Lab 2	38.30	Lab 2	39.25	
Sorted, x_i	Lab 5	38.75	Lab 1	40.75	
…	Lab 1	38.88	Lab 5	43.38	$S = \sqrt{\frac{1}{p_j - 1}\sum_{i=1}^{p_j}(x_i - \bar{x})^2}$
x_p=	Lab 3	47.75	Lab 3	49.50	
$\bar{x}$	37.42		40.00		
S	5.482		5.165		
G_1	1.034		0.9680		
Critical 1%	2.139		2.139		$G_1 = \frac{(\bar{x} - x_1)}{S}$
Critical 5%	2.020		2.020		

Acceptable * Stragglers ** Outliers

G values 5% 1%

Grubb's single low test: All mean data acceptable.

Grubb's test, for between-laboratory / between-operator consistency. Test double highest mean.

Single High Observation	Structural Element				
	Wall 1		Slab 1		…..j….q
	lab/opr	$\bar{y}_{ij}$	lab/opr	$\bar{y}_{ij}$	
x_1 =	Lab 7	31.75	Lab 6	35.00	$\bar{x}_{p-1,p} = \frac{1}{p_j - 2}\sum_{i=1}^{p_j - 2} x_i$
…	Lab 6	32.25	Lab 7	35.25	
…	Lab 4	34.25	Lab 4	36.88	
…	Lab 2	38.30	Lab 2	39.25	
Sorted, x_i	Lab 5	38.75	Lab 1	40.75	$S^2_{p-1,p} = \sum_{i=1}^{p_j - 2}(x_i - \bar{x}_{p-1,p})^2$
…	Lab 1		Lab 5		
x_p =	Lab 3		Lab 3		
$\bar{x}_{p-1,p}$	35.06		37.42		
$S^2_{p-1,p}$	43.62		25.30		$S^2_o = \sum_{i=1}^{p_j}(x_i - \bar{x})^2$
S^2_o	180.3		160.1		
G_{2p}	0.2420		0.1580		$G_{2p} = \frac{S^2_{p-1,p}}{S^2_o}$
Critical 1%	0.0308		0.0308		
Critical 5%	0.0708		0.0708		

** Outliers | * Stragglers | Acceptable

G values | 1% | 5%

Grubb's double high test: All mean data acceptable.

Grubb's test, for between-laboratory / between-operator consistency. Test double lowest mean.

Single High Observation	Structural Element				
	Wall 1		Slab 1		
	lab/opr	$\overline{y}_{ij}$	lab/opr	$\overline{y}_{ij}$	.....j....q
x_1 =	Lab 7		Lab 6		$\overline{x}_{1,2} = \frac{1}{p_j - 2}\sum_{i=3}^{p_j} x_i$
…	Lab 6		Lab 7		
…	Lab 4	34.25	Lab 4	36.88	
…	Lab 2	38.30	Lab 2	39.25	
Sorted, x_i	Lab 5	38.75	Lab 1	40.75	$S^2{}_{1,2} = \sum_{i=3}^{p_j}\left(x_i - \overline{x}_{1,2}\right)^2$
…	Lab 1	38.88	Lab 5	43.38	
x_p =	Lab 3	47.75	Lab 3	49.50	
$\overline{x}_{1,2}$	39.58		41.95		
$S^2{}_{1,2}$	97.98		93.52		$S^2{}_o = \sum_{i=1}^{p_j}\left(x_i - \overline{x}\right)^2$
$S^2{}_o$	180.3		160.1		
$G_{1,2}$	0.5434		0.5842		
Critical 1%	0.0308		0.0308		$G_{1,2} = \frac{S^2{}_{1,2}}{S^2{}_o}$
Critical 5%	0.0708		0.0708		

** Outliers | * Stragglers | Acceptable

G values | 1% | 5%

Grubb's double low test: All mean data acceptable.

B4.6. Calculation of Mean, Repeatability and Reproducibility

Submission Lab or Operator	Structural Element								
	Wall 1				Slab 1				…..j….q
	$\bar{y}_{ij}$	S_{ij}	S_{ij}^2	n_{ij}	$\bar{y}_{ij}$	S_{ij}	S_{ij}^2	n_{ij}	
Lab 1	38.88	0.4787	0.2292	4	40.75	0.2887	0.08333	4	
Lab 2	38.30	0.6055	0.3667	4	39.25	0.2887	0.08333	4	
Lab 3	47.75	0.5000	0.2500	4	49.50	0.5774	0.3333	4	
Lab 4	34.25	0.5000	0.2500	4	36.88	1.931	3.729	4	
Lab 5	38.75	0.5000	0.2500	4	43.38	1.181	1.396	4	
Lab 6	32.25	0.9574	0.9167	4	35.00	1.155	1.333	4	
Lab 7	31.75	0.9574	0.9167	4	35.25	0.5000	0.2500	4	
. i . p									

	Wall 1	Slab 1	
p_j	7	7	
General mean, $\hat{m}_j = \bar{\bar{y}}_j$	37.42	40.00	$\hat{m}_j = \bar{\bar{y}}_j = \dfrac{\sum_{i=1}^{p_j}\left(n_{ij}\times \bar{y}_{ij}\right)}{\sum_{i=1}^{p_j} n_{ij}}$
Repeatability variance, S_{rj}^2	0.4542	1.030	$S_{rj}^2 = \dfrac{\sum_{i=1}^{p_j}\left[(n_{ij}-1)xS_{ij}^2\right]}{\sum_{i=1}^{p_j}(n_{ij}-1)}$
$\bar{\bar{n}}_j$	4.0	4.0	$\bar{\bar{n}}_j = \dfrac{1}{p_j-1}\left(\sum_{i=1}^{p_j} n_{ij} - \dfrac{\sum_{i=1}^{p_j} n_{ij}^2}{\sum_{i=1}^{p_j} n_{ij}}\right)$
S_{dj}^2	120.2	106.7	$S_{dj}^2 = \dfrac{1}{p_j-1}\left[\sum_{i=1}^{p_j}\left(n_{ij}\bar{y}_{ij}^2\right) - \bar{\bar{y}}_j^2 \sum_{i=1}^{p_j} n_{ij}\right]$
Between lab/opr variance, S_{Lj}^2	29.94	26.42	$S_{Lj}^2 = \dfrac{S_{dj}^2 - S_{rj}^2}{\bar{\bar{n}}_j}$
Reproducibility variance, S_{Rj}^2	30.39	27.45	$S_{Rj}^2 = S_{rj}^2 + S_{Lj}^2$
S_{Rj}	5.513	5.240	
S_{rj}	0.6739	1.015	

B4.7. Calculation of Measurement Uncertainty

B4.7.1 Proficiency Test Data

From the proficiency test data, the repeatability standard deviation of the test method is estimated as 0.6739 (Wall 1) and 1.015 (Slab 1) and the reproducibility standard deviation of the test method is estimated as 5.513 (Wall 1) and 5.240 (Slab 1).

B4.7.2 Control of Bias

For a laboratory to show sufficient evidence of bias control, the standard deviation for proficiency testing has to be less than S_R and the laboratory has a mean z-score between $\pm\frac{2}{\sqrt{q}}$ for q assigned value.

Where excessive bias is detected, action will have to be taken to bring the bias within the required range before proceeding with measurements. Such action will involve investigation and elimination of the cause of the bias.

B4.7.3 Control of Precision

Laboratory has to demonstrate that its repeatability standard deviation is within the range found in the proficiency test. When this is the case, the precision is accordingly considered to be under good control.

B4.7.4 Measurement Uncertainty

The uncertainty $u(y)$ associated with an observation can be estimated using the following equation:

$$u^2(y) = u^2(\delta) + s_R^2 + \sum c_i^2 u_i^2(x_i)$$

where,

$$u(\delta) = s_\delta = \sqrt{\frac{s_R^2 - (1-\frac{1}{n})s_r^2}{p}}$$

$\sum c_i^2 u_i^2(x_i)$ = 0 (assuming that the controlling variables during the proficiency testing and routine testing remain constant)

For Wall 1

$$u(\delta) = s_\delta = \sqrt{\frac{s_R^2 - (1-\frac{1}{n})s_r^2}{p}} = \sqrt{\frac{30.39 - (1-\frac{1}{4})0.4542}{7}} = 2.072$$

Therefore,

$$u^2(y) = 2.072^2 + 5.513^2 = 34.69$$

$$u(y) = 5.890$$

For Slab 1

$$u(\delta) = s_\delta = \sqrt{\frac{s_R^2 - (1-\frac{1}{n})s_r^2}{p}} = \sqrt{\frac{27.45-(1-\frac{1}{4})1.030}{7}} = 1.952$$

Therefore,

$$u^2(y) = 1.952^2 + 5.240^2 = 31.27$$

$$u(y) = 5.592$$

B4.7.5 Expanded Uncertainty

No. of test results per laboratory = 4

Total no. of test results per structural element in this proficiency study = 28

Degree of freedom, $v = 28 - 1 = 27$

From the Student's t table and for 95% Confidence Interval, the coverage factor k = 2.052 (from interpolation)

For Wall 1

The expanded uncertainty, $U(y) = ku(y)$

$$= 2.052 \times 5.890$$

$$= 12.09$$

For Slab 1

The expanded uncertainty, $U(y) = ku(y)$

$$= 2.052 \times 5.592$$

$$= 11.47$$

Therefore, the measurement uncertainty in the RH measurement on Wall 1 and Slab 1 is 12.1 and 11.5 respectively at 95% confidence level.

EXAMPLE (B5) – ROCKWELL 'C' HARDNESS OF METALLIC SAMPLE

B5.1. Introduction

This example serves to illustrate the estimation of measurement uncertainty according to ISO/TS 21748 approach. Repeatability and reproducibility information as required by the approach are obtained from proficiency testing data and calculated based on ISO 5725-2 (shown below from step B5.2 to step B5.6). The proficiency testing data is obtained from Rockwell C hardness measurement of metallic sample tested in accordance to ASTM E18-05.

Definition*:*

i	laboratory or operator	x	data set using mean
j	level, e.g. targeted hardness of sample	y	individual results
k	individual results by i	$\bar{y}$	mean of y of the laboratory or operator
p	no. of laboratory or operator	$\bar{\bar{y}}$	general mean of the level
q	no. of levels	s	spread of results, e.g. standard deviation
n	no. of results by each laboratory or operator		

B5.2. Original Data

Laboratory	Level	
	44 HRC	…..j….q
Lab 1	43.6	
	43.5	
	43.9	
	43.2	
	43.2	
	43.6	
	43.7	
	43.5	
	43.6	
	43.6	
	43.7	
	43.5	
Lab 2	43.2	
	43.2	
	43.2	
	43.2	
	43.2	
	43.2	
	43.0	
	43.0	
	43.4	
	43.2	
	43.2	
	43.2	
Lab 3	42.0	
	42.6	
	42.9	
	43.2	
	42.9	
	43.3	
	43.1	

	43.2	
	43.1	
	43.2	
	43.3	
	43.4	
Lab 4	43.4	
	44.0	
	43.8	
	44.0	
	44.1	
	43.9	
	43.8	
	43.9	
	44.1	
	43.8	
	44.1	
	43.8	
. . i . p		y_{ij1} , k=1 … y_{ijk} … $y_{ijn_{ij}}$, k=n_{ij}

B5.3. Mean for each Laboratory or Operator

Submission Lab or Operator	**Level**		
	44 HRC		…..j….q
	$\overline{y_{ij}}$	n_{ij}	
Lab 1	43.6	12	
Lab 2	43.2	12	
Lab 3	43.0	12	
Lab 4	43.9	12	
. i . p			$\overline{y_{ij}} = \frac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} y_{ijk}$
General mean, $\widehat{m}_j = \overline{\overline{y_j}}$	43.41		$\widehat{m}_j = \overline{\overline{y_j}} = \frac{\sum_{i=1}^{p_j} \left(n_{ij} \times \overline{y_{ij}}\right)}{\sum_{i=1}^{p_j} n_{ij}}$

Note: Express to 1 more significant digit than original data

B5.4. Spread of each Cell (commonly sample standard deviation)

Submission Lab or Operator	**Level**		
	44 HRC		…..j….q
	S_{ij}	S_{ij}^2	
Lab 1	0.198	0.0391	
Lab 2	0.103	0.0106	
Lab 3	0.388	0.1501	
Lab 4	0.198	0.0391	

. i . p			$S_{ij} = \sqrt{\frac{1}{n_{ij}-1}\sum_{k=1}^{n_{ij}}\left(y_{ijk} - \overline{y_{ij}}\right)^2}$
$\sum_{i=1}^{p} S_{ij}^2$		0.2393	

Note: Express to 1 more significant digit than original data

B5.5. Scrutiny of Results for Consistency and Outliers

- Graphical Method

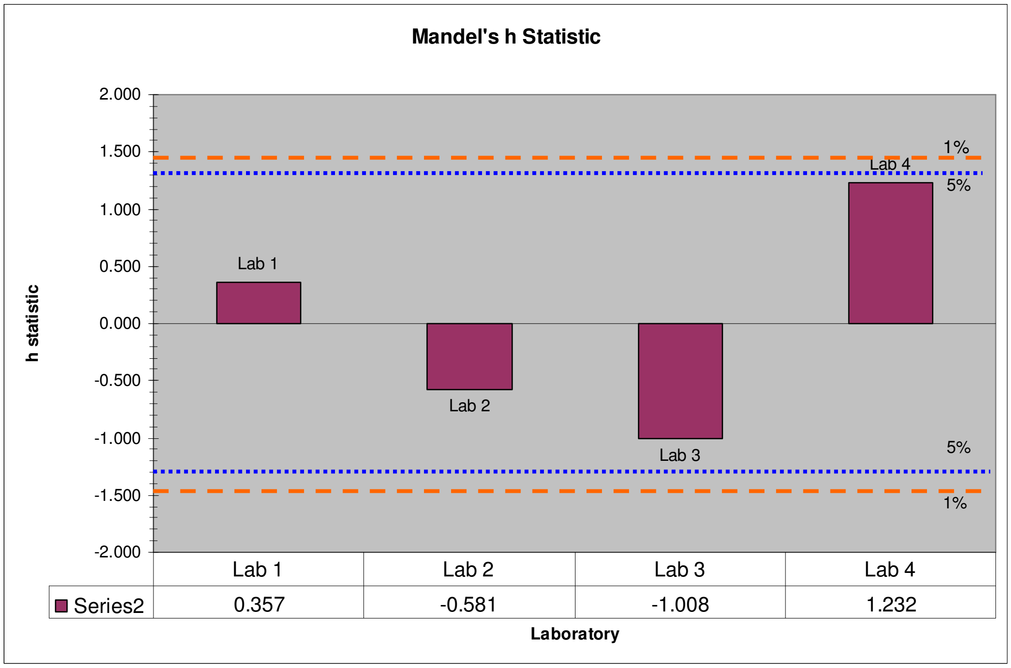
Mandel's h statistics, for between-laboratory / between-operator consistency

Submission Lab or Operator	**Level**	
	44 HRC	…..j….q
Lab 1	0.357	
Lab 2	-0.581	
Lab 3	-1.008	
Lab 4	1.232	
. i . p		$h_{ij} = \frac{\overline{y_{ij}} - \overline{\overline{y_j}}}{\sqrt{\frac{1}{p_j-1}\sum_{i=1}^{p_j}\left(\overline{y_{ij}} - \overline{\overline{y_j}}\right)^2}}$
p_j	4	
1% Significance Level	1.49	
5% Significance Level	1.42	

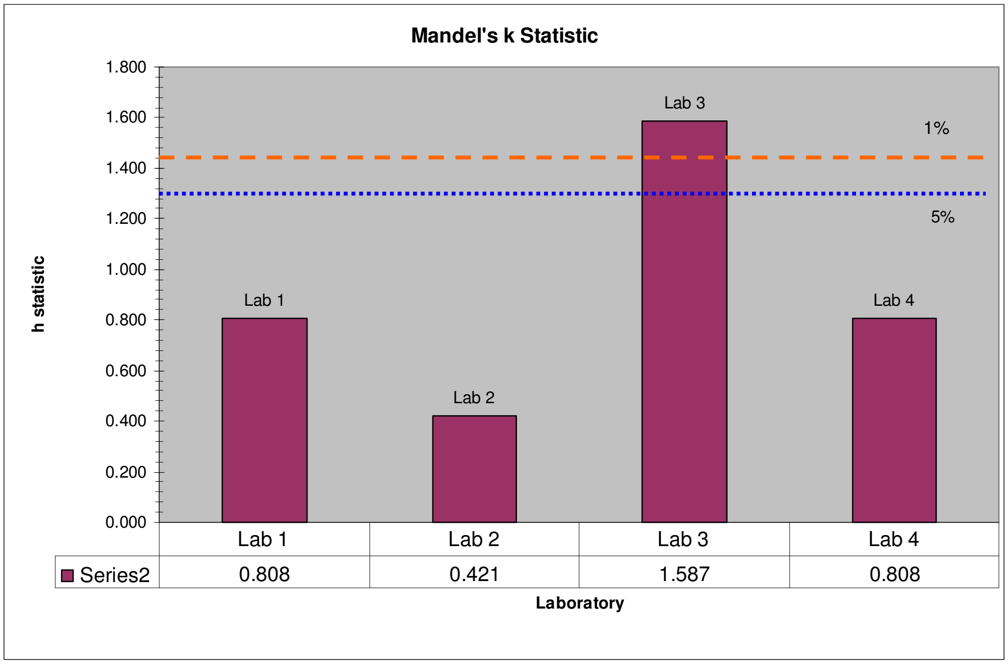
Mandel's k statistics, for within-laboratory / operator consistency

Submission Lab or Operator	**Level**	
	44 HRC	…..j….q
Lab 1	0.808	
Lab 2	0.421	
Lab 3	1.587	**
Lab 4	0.808	
. i . p		$k_{ij} = \frac{S_{ij}\sqrt{p_j}}{\sqrt{\sum_{i=1}^{p_j} S_{ij}^2}}$
p_j	4	n_j = no. of test results occurring in majority of the cells
n_j	12	
1% Significance Level	1.43+	+ *: Based on n = 10*
5% Significance Level	1.31+	+ *: Based on n = 10*

Mandel's h Test: No outlier / straggler at Level 44 HRC.
Mandel's k Test: Outlier at Level 44 HRC, Lab 3



Mandel's h Statistic
h statistic
2.000
1.500
1.000
0.500
0.000
-0.500
-1.000
-1.500
-2.000
Lab 1
Lab 2
Lab 3
Lab 4
1%
5%
5%
1%
Series2
0.357
-0.581
-1.008
1.232
Laboratory

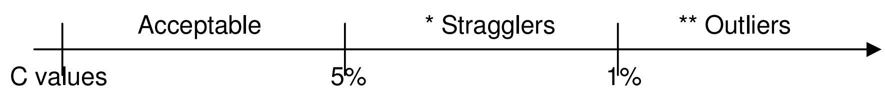


Mandel's k Statistic
h statistic
1.800
1.600
1.400
1.200
1.000
0.800
0.600
0.400
0.200
0.000
Lab 1
Lab 2
Lab 3
Lab 4
1%
5%
Series2
0.808
0.421
1.587
0.808
Laboratory

- Numerical Method

Cochran's test, for within laboratory / operator consistency. Test only the highest spread.

Submission Lab or Operator	Level		
	44 HRC		…..j….q
	$S^2_{ij,\,max}$	C	
Lab 1			
Lab 2			
Lab 3	0.150	0.6268	**
Lab 4			
. i . p			$C = \dfrac{S^2_{ij,\,max}}{\sum_{i=1}^{p_j} S_{ij}^2}$
p_j	4		
n_j	12		
Critical 1%	0.5536+		*+Based on n = 10*
Critical 5%	0.4884+		*+Based on n = 10*



Cochran's test: Lab 3 result is an outlier.

Remove Lab 3 results and repeat Cochan's test.

Mean for each Laboratory or Operator (Lab 3 rejected as Outlier)

Submission Lab or Operator	Level		
	44 HRC		…..j….q
	$\overline{y}_{ij}$	n_{ij}	
Lab 1	43.6	12	
Lab 2	43.2	12	
Lab 3	-	-	
Lab 4	43.9	12	
. i . p			$\overline{y}_{ij} = \dfrac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} y_{ijk}$
General mean, $\hat{m}_j = \overline{\overline{y}}_j$	43.54		$\hat{m}_j = \overline{\overline{y}}_j = \dfrac{\sum_{i=1}^{p_j} \left(n_{ij} \times \overline{y_{ij}}\right)}{\sum_{i=1}^{p_j} n_{ij}}$

Note: Express to 1 more significant digit than original data

Spread of each Cell (commonly sample standard deviation) (Lab 3 rejected as Outliers)

Submission Lab or Operator	Level		
	44 HRC		…..j….q
	S_{ij}	S_{ij}^2	
Lab 1	0.198	0.0391	
Lab 2	0.103	0.0106	

Lab 3	-	-	
Lab 4	0.198	0.0391	
. i . p			$S_{ij} = \sqrt{\frac{1}{n_{ij}-1}\sum_{k=1}^{n_{ij}}\left(y_{ijk} - \overline{y_{ij}}\right)^2}$
$\sum_{i=1}^{p} S_{ij}^2$		0.0888	

Note: Express to 1 more significant digit than original data

Recalculation of Cochan's Test (without Lab 3)

Submission Lab or Operator	**Level**		
	44 HRC		…..j….q
	$S^2_{ij,\max}$	C	
Lab 1	0.0391	0.4403	**Acceptable**
Lab 2			
Lab 4	0.0391	0.4403	**Acceptable**
. i . p			$C = \frac{S^2_{ij,\max}}{\sum_{i=1}^{p_j} S_{ij}^2}$
p_j	3		
n_j	12		
Critical 1%	0.6743⁺		⁺*Based on n = 10*
Critical 5%	0.6167⁺		⁺*Based on n = 10*

Cochran's test: Lab 1 and 4 results are acceptable.

Grubb's Test

Grubb's Test: Check single high and single low mean.
If no outlier, then check double high and double low mean.

Grubb's test, for between-laboratory/between-operator consistency. Test single highest mean.

Single High Observation	**Level**		
	44 HRC		…..j….q
	lab/opr	$\overline{y_{ij}}$	
x_1 =	Lab 2	43.18	$\overline{x} = \frac{1}{p_j}\sum_{i=1}^{p_j} x_i$
Sorted, x_i	Lab 1	43.55	
x_p =	Lab 4	43.89	
$\overline{x}$	43.540		
S	0.354		
G_p	0.988		
Critical 1%	1.155		

Critical 5%	1.155	$S = \sqrt{\frac{1}{p_j - 1}\sum_{i=1}^{p_j}\left(x_i - \bar{x}\right)^2}$ $G_p = \frac{\left(x_p - \bar{x}\right)}{S}$

Acceptable | * Stragglers | ** Outliers

G values | 5% | 1%

Grubb's single high test: The highest mean is acceptable.

Grubb's test for between-laboratory / between-operator consistency. Test single lowest mean.

Single Low Observation	Level		
	44 HRC		…..j….q
	lab/opr	$\bar{y}_{ij}$	
x_1=	Lab 2	43.18	$\bar{x} = \frac{1}{p_j}\sum_{i=1}^{p_j} x_i$
Sorted, x_i	Lab 1	43.55	
x_p=	Lab 4	43.89	
$\bar{x}$	49.540		$S = \sqrt{\frac{1}{p_j - 1}\sum_{i=1}^{p_j}\left(x_i - \bar{x}\right)^2}$
S	0.354		
G_1	1.012		$G_1 = \frac{\left(\bar{x} - x_1\right)}{S}$
Critical 1%	1.155		
Critical 5%	1.155		

Acceptable | * Stragglers | ** Outliers

G values | 5% | 1%

Grubb's single low test: All mean data acceptable.

Double high and double low mean not checked as insufficient number of labs.

B5.6. Calculation of Mean, Repeatability and Reproducibility

Submission Lab or Operator	Level				
	44 HRC				…..j….q
	$\bar{y}_{ij}$	S_{ij}	S_{ij}^2	n_{ij}	
Lab 1	43.6	0.198	0.0391	12	
Lab 2	43.2	0.103	0.0106	12	
Lab 3	-	-	-	-	
Lab 4	43.9	0.198	0.0391	12	
. i . p					
p_j	3				

General mean, $\hat{m}_j = \bar{\bar{y}}_j$	43.54	$\hat{m}_j = \bar{\bar{y}}_j = \dfrac{\sum_{i=1}^{p_j}\left(n_{ij} \times \overline{y_{ij}}\right)}{\sum_{i=1}^{p_j} n_{ij}}$
Repeatability variance, S_{rj}^2	0.027	$S_{rj}^2 = \dfrac{\sum_{i=1}^{p_j}\left[(n_{ij}-1) x S_{ij}^2\right]}{\sum_{i=1}^{p_j}(n_{ij}-1)}$
$\bar{\bar{n}}_j$	12.0	$\bar{\bar{n}}_j = \dfrac{1}{p_j - 1}\left(\sum_{i=1}^{p_j} n_{ij} - \dfrac{\sum_{i=1}^{p_j} n_{ij}^2}{\sum_{i=1}^{p_j} n_{ij}}\right)$
S_{dj}^2	1.51	$S_{dj}^2 = \dfrac{1}{p_j - 1}\left[\sum_{i=1}^{p_j}\left(n_{ij}\, \bar{y}_{ij}^{\,2}\right) - \bar{\bar{y}}_j^{\,2} \sum_{i=1}^{p_j} n_{ij}\right]$
Between lab/opr variance, S_{Lj}^2	0.12	$S_{Lj}^2 = \dfrac{S_{dj}^2 - S_{rj}^2}{\bar{\bar{n}}_j}$
Reproducibility variance, S_{Rj}^2	0.15	$S_{Rj}^2 = S_{rj}^2 + S_{Lj}^2$
S_{Rj}	0.39	
S_{rj}	0.16	

Repeatability = $S_r = \dfrac{1}{q}\sum_{j=1}^{q} S_{rj}$ **= 0.16 HRC**

Reproducibility = $S_R = \dfrac{1}{q}\sum_{j=1}^{q} S_{Rj}$ **= 0.39 HRC**

B5.7. Comments

Precision and Bias from Test Method

ASTM E18-05 – Section 10

Table 1 Precision data for measurements of the Rockwell C hardness test block (45.0 HRC).

Ave Hardness	**Repeatability Conditions**		**Reproducibility Conditions**	
	s_r	**r**	**s_R**	**R**
45.35 HRC	0.156	0.438	0.259	0.725

B5.8. Calculation of Measurement Uncertainty

B5.8.1 Proficiency Test Data

From the proficiency test data, the repeatability standard deviation of the test method is estimated as 0.16 HRC and the reproducibility standard deviation of the test method is estimated as 0.39 HRC.

B5.8.2 Control of Bias

For a laboratory to show sufficient evidence of bias control, the standard deviation for proficiency testing has to be less than S_R and the laboratory has a mean z-score between $\pm \frac{2}{\sqrt{q}}$ for q assigned value.

Where excessive bias is detected, action will have to be taken to bring the bias within the required range before proceeding with measurements. Such action will involve investigation and elimination of the cause of the bias.

B5.8.3 Control of Precision

Laboratory has to demonstrate that its repeatability standard deviation is within the range found in the proficiency test. When this is the case, the precision is accordingly considered to be under good control.

B5.8.4 Measurement Uncertainty

The uncertainty $u(y)$ associated with an observation can be estimated using the following equation:

$$u^2(y) = u^2(\delta) + s_R^2 + \sum c_i^2 u_i^2(x_i)$$

where,

$$u(\delta) = s_\delta = \sqrt{\frac{s^2{}_R - (1-1/n)s^2{}_r}{p}} = \sqrt{\frac{0.15-(1-1/12)0.027}{3}}$$

$$= 0.204$$

$\sum c_i^2 u_i^2(x_i)$ = 0 (assuming that the controlling variables during the proficiency testing and routine testing remain constant)

Therefore,

$$u^2(y) = 0.204^2 + 0.39^2 = 0.19$$

$$u(y) = 0.44$$

B5.8.5 Expanded Uncertainty

No. of test results per laboratory = 12

Total no. of test results in this proficiency study = 36

Degree of freedom, $v = 36 - 1 = 35$

From the Student's t table and for 95% Confidence Interval, the coverage factor k = 2.03 (from interpolation)

The expanded uncertainty, $U(y) = ku(y)$

$$= 2.03 \times 0.44$$

$$= 0.89 HRC$$

Therefore, the measurement uncertainty in the Rockwell C hardness is 0.89 HRC at 95% confidence level.

APPENDIX C

SIGNIGIFICANT DIGITS IN RESULTS FROM MEASUREMENT

C1. Introduction

C1.1 Measurements are made by reading calibrated scales or digital displays. The last significant digits of the measurement are the precision of the measuring tool e.g. a meter rule is scaled into millimeter division equivalent to 1/1000 of a meter. If there is no finer sub-division indicated, the meter rule maybe read to a precision of 0.5mm, this being an estimate between the smallest division on the scale. Measurements cannot be made finer than the precision of the scale, any numeric digits written further to the right of this is therefore not significant.

For example, the average of 3 readings: 150.5mm, 151.5mm and 148.5mm, measured using the same meter rule is 150.167mm. This is therefore significant up to the first decimal place, 0.2mm, as the individual readings are significant up to the first decimal place only. The result of the average is therefore expressed as 150.2mm (it is worth noting that the uncertainty of the average is +/-1.0mm).

C1.2 For digital display, the last non-fluctuating digit in the readout is the precision of the reading. The last significant digit of the measurement is therefore the larger of the precision of the reading and the precision of the measuring tool.

For example: an electronic weighing scale has a precision of 100grams but the display could be read without fluctuation to about 8320grams. While the precision of the reading is 10grams, the last significant digit of the measurement is 100grams, that is, 8300grams - in line with the larger of the precisions, that is, that of the measurement tool. The result of the weight measurement therefore has 2 significant digits; 8300grams.

C2. Expression of Significant Digits

C2.1 If in a similar weighing exercise above, a measurement is written as 8000grams, the result could mean significance up to a 1000grams, 100grams, 10 grams or 1grams, that is 4, 3, 2, or 1 significant digits. This shows that the significance of the digit zeros is unclear. The following guide may be useful:

1) All non-zero digits are always significant.
2) Zeros before any other digits are not significant, e.g. 0.089gram has 2 significant digits (it is worth noting that precision in this reading is 0.001gram)
3) Zeros placed after a decimal point following a digit are always significant, e.g. 8003.50grams has 6 significant digits (indicating precision to 0.01gram)
4) Zeros between other digits are always significant, e.g. 8003grams has 4 significant digits
5) Zeros at the end of a number without decimal points, e.g. 8300grams, are only sometimes significant.

C2.2 To avoid this ambiguity, it is recommended that the results be expressed in scientific notation:

8×10^3grams to indicate	1 significant digit
8.0×10^3grams	2 significant digits
8.00×10^3grams	3 significant digits
8.000×10^3grams	4 significant digits

C3. Propagation of Significant Digits in Mathematical Operations

C3.1 When performing any mathematical operation, it is important to remember that the results can never be more precise than the least precise measurement.

C3.2 For addition and subtraction: first perform the arithmetic operation, then round off the results to correspond to the least precise value involved.

For example: 150.15mm + 1151.5mm + 148.523mm = 1450.173mm is rounded off to 1450.2mm in line with the least precise value of 1151.5mm (a precision of 1 decimal place).

C3.3 For multiplication and division: first perform the arithmetic operation, then round off the product or quotient to the same number of significant digits as the factors with the least significant digits.

For example: area = 150.1mm $\times$ 1151.5mm = 172840.15mm^2 is round off to 172800mm^2 in line with the least significant digits of 150.1mm.

C3.4 In evaluating mathematical functions such as trigonometric and exponential, the results should yield the same significant digits as the factors with the least significant digits.

For example: sin(0.097m^{-1} $\times$ 4.73m) = 0.44288 is rounded off to 0.44 in line with the 2 significant digits of 0.097m^{-1}.

C3.5 Only results of measurement have uncertainty and hence the need to express results to the relevant significant digits. Numerical constants with exact definition, e.g. conversion factors etc, and numbers arising from counting do not have associated uncertainty and hence may be thought to have infinite significant digits and precision.

For example: the average of 3 readings: 150.15mm, 151.5mm and 148.523mm is calculated as (150.15mm + 151.5mm + 148.523mm) $\div$ 3. The counting number of "3" has infinite significant digits therefore the answer of this quotient is limited by the least significant digits in the numerators, in this case, 4 from the measurement 151.5mm.

C3.6 For numeric constants without exact definition, such as π = 3.141592654, density of water, gravitation g… etc, it is recommended to express this value in significant digits commensurate with the least significant digits of the rest of the computation.

C4. Keeping an extra digit in Intermediate Answer

C4.1 When performing multiple-step calculation, it is recommended to keep at least one more digit in excess of the last significant digit. This is to avoid accumulation of rounding off errors in the calculation. It is recommended that the same number of extra digits be maintained throughout the calculation for ease of truncation to the correct significant digits in the final answer. Refer to attached example where an extra digit is kept consistently for all intermediate calculations and truncated only in the final answer.

C4.2 In mathematical operations which involves several arithmetic procedures (e.g. $(A-B)/(C-D)$) propagation of significant digits should be noted at every stage however truncation, to the relevant significant digits, should only be performed at the end of the operation.

C5. Rounding off numbers

C5.1 In measurement or mathematical operations, digits in excess of the last significant digit are termed superfluous digits. These superfluous digits are rounded off if:

1) it is less than 5, the last significant digit remains unchanged
2) it is more than 5, the last significant digit is increased by 1
3) it is exactly equal to 5, the last significant digit is rounded off to be an even number (in accordance to ASTM E29)

C5.2 In some measurement it may be required to round off the results to the nearest x, (e.g. to the nearest 0.5), the procedure from ASTM E29 may be adopted :

1) First divide the number by x, (in the case of 0.5, this is similar to multiplying the number by 2)
2) Round off the results from (1) to an integer, based on the steps given in C5.1 above
3) Multiple the results from (2) by x, (in the case of 0.5, this is similar to dividing the number by 2)

For example, rounding 43.75MPa to the nearest 0.5MPa, would yield 44.0MPa.

Working:	1)	dividing by 0.5,	43.75 / 0.5 = 87.5,
	2)	rounding off to an integer	= 88.0,
	3)	multiplying by 0.5,	88.0 x 0.5 = 44.0

For example, rounding 43.75MPa to the nearest 0.3MPa, would yield 43.8MPa.

Working:	1)	dividing by 0.3,	43.75 / 0.3= 145.83,
	2)	rounding off to an integer	= 146.0,
	3)	multiplying by 0.3,	146.0 x 0.3 = 43.8

EXAMPLE (C1) – COMPRESSIVE STRENGTH OF CONCRETE CUBE

C1.1 Introduction

This example serves to illustrate the propagation of significant digits in the estimation of measurement uncertainty according to Guide to the Expression of Uncertainty in Measurement (GUM: 1995) in the determination of compressive strength of cubes tested to SS78: Part A16: 1987.

The working uses nominal uncertainties and recommended equipment tolerance limits as specified in the test method.

Explanatory notes are included in italics. Note: sign digits = significant digits.

C1.2 Model

Cube compressive strength, $f_{cu} = \frac{F}{A}$, …(1)

where F = maximum applied load on cube, and
A = cross sectional area,
= L_{avg}^2 …(2)
L_{avg} = average of 2 pairs of orthogonal dimensions perpendicular to direction of loading

C1.3 Measurement

Max. load, F =1119.4kN	Nominal dimension, L_{avg} (mm)	Area, A (mm^2)	Strength, f_{cu} (N/mm^2)
1119400N =$1.1194x10^6$N	150	22500=$2.25x10^4$	49.751… =49.8
5 sign digits	*3 sign digits*	*3 sign digits*	*3 sign digits*

C1.4 Sources of uncertainty

a) Force measurement — Test method recommends using machine of accuracy ±1% of indicated load, SS78: Part A15: 1987

b) Dimension measurement — Within 1% variation in cube nominal dimensions is permissible, SS78: Part A14: 1987

c) Sampling — as received sample, thus sampling uncertainty not included in this computation.

C1.5 Estimation of standard uncertainty from major components

a) Force measurement:

Accuracy of testing machine = 1%, assume rectangular dist.

Relative standard uncertainty $= \dfrac{\text{Est.U}}{\sqrt{3}} = \dfrac{1\%}{\sqrt{3}} =$ 0.58%

Standard uncertainty, u_F $= 0.58\% \times 1.1194\text{x}10^6\text{N}$
$= 6.5\text{x}10^3\text{N}$

"Est.U" has 1 sign digit therefore answer is rounded to 1+1 sign digit, the additional significant digits is kept for intermediate working only.

Type B evaluation
Degree of freedom, ν_F $= \propto$

b) Length measurement

Permissible variation in dimension= 1%, assume rectangular dist.

Relative standard uncertainty $= \dfrac{\text{Est.U}}{\sqrt{3}} = \dfrac{1\%}{\sqrt{3}} =$ 0.58%

Standard uncertainty, u_L $= 0.58\% \times 150 =$ 0.87mm

"Est.U" has 1 sign digit therefore answer is rounded to 1+1 sign digit

Type B evaluation
Degree of freedom, ν_L $= \propto$

C1.6 ***Combined standard uncertainty of the major components***

From Equation (1):

Sensitivity coefficient of force, c_F $= \dfrac{\partial f_{cu}}{\partial F} = \dfrac{1}{A}$

$= \dfrac{1}{2.25\text{x}10^4} = 4.444\text{x}10^{-5}/\text{mm}^2$

"A" has 3 sign digits therefore answer is rounded to 3+1 sign digits

Sensitivity coefficient of area, c_A $= \dfrac{\partial f_{cu}}{\partial A} = -\dfrac{F}{A^2}$

$= -\dfrac{1.1194\times10^6}{(2.25\times10^4)^2} = 2.211\text{x}10^{-3}\text{N/mm}^4$

"A" has 3 sign digits while "F" has 5 sign digits therefore answer is rounded to 3+1 sign digits – least of sign digits in "A" or "F"

From Equation (2):

Sensitivity coefficient of length, c_L $= \dfrac{\partial A}{\partial L_{avg}} = 2\times L_{avg}$

$= 2\times150 = 300.0\text{mm} = 3.000\text{x}10^2\text{mm}$

"L_{avg}" has 3 sign digits therefore answer is rounded to 3+1 sign digits
The combined standard uncertainty of area, u_A

$u_A = \sqrt{c_L{}^2 \times u_L{}^2} = \sqrt{(3.0000\times10^2)^2\times0.87^2}$ = 261mm^2= 2.6x10^{2}mm^2

"c_L" has 3+1 sign digits while "u_L" has 1+1 sign digit therefore answer is rounded to 1+1 sign digit – least of sign digit in "c_L" or "u_L"

Therefore, the combined standard uncertainty of strength, u_c

$$u_c = \sqrt{c_F{}^2 \times u_F{}^2 + c_A{}^2 \times u_A{}^2}$$
$$= \sqrt{(4.444\times10^{-5})^2\times(6.5\times10^3)^2 + (2.211\times10^{-3})^2\times(2.6\times10^2)^2}$$

= 0.64335N/mm^2
= 0.64N/mm^2

"c_F" and "c_A" has 3+1 sign digits while "u_F" and"u_A" has 1+1 sign digit therefore answer is rounded to 1+1 sign digit – least of sign digit in "c_F", "c_A", "u_F" or "u_A"

C1.7 Estimate of expanded uncertainty

Coverage factor,

Effective degree of freedom of area, ν_A

$$\nu_A = \frac{u_A{}^4}{\sum_{i=1}^{N}\frac{c_L{}^4 u_{L,i}{}^4}{\nu_{L,i}}} = \frac{(2.6\times10^2)^4}{\left(\frac{(3.000\times10^2)^4\times0.87^4}{\propto}\right)} = \infty$$

Effective degree of freedom of strength, ν_{eff}

$$\nu_{eff} = \frac{u_c{}^4}{\sum_{\substack{i=1\\j=1}}^{N}\frac{c_i{}^4 u_{i,j}{}^4}{\nu_{i,j}}}$$

$$= \frac{0.64^4}{\left(\frac{(4.444\times10^{-5})^4\times(6.5\times10^3)^4}{\propto} + \frac{(2.211\times10^{-3})^4\times(2.6\times10^2)^4}{\propto}\right)} = \infty$$

k = 1.96 at 95% level of confidence

"k"is given as 3 sign digits, 1 more sign digit than "u_C"

Expanded uncertainty, U= $k\,u_c$
= 1.96 × 0.64N/mm^2 = 1.25N/mm^2 = 1.3N/mm^2

"k" has 3 sign digits while "u_C" has 1+1 sign digit therefore answer is rounded to 1+1 sign digit – least of sign digit in "k" or "u_C"

The cube compressive strength, f_{cu} = 49.8N/mm^2 while the measurement uncertainty is ± 1.3N/mm^2, inclusive of 1+1 intermediate sign digit

Therefore, the cube compressive strength = 49.8 ± 1.3N/mm^2

C1.8 Reporting of results

The cube compressive strength, f_{cu} = 50 ± 1N/mm^2 at level of confidence of 95% (k=1.96)

After removal of the intermediate sign digit and rounding off the primary result to similar precision

Therefore the cube compressive strength is 50.0N/mm^2 tested in accordance to SS78:Part A16:1987

Reported result is rounded off to nearest 0.5 N/mm^2 as required by test method but written without corresponding measurement uncertainty.

APPENDIX D

EXPANDED UNCERTAINTY AND COVERAGE FACTOR

D1. Introduction

A useful expression of the combine standard uncertainty is to define an interval about the results within which the value of the measurand can be confidently said to lie. More specifically, an interval $y - u_p \leq Y \leq y + u_p$, which is commonly written as $Y = y \pm u_p$, having an approximate level of confidence p that the value of Y will be within the interval $y \pm u_p$. The interval reflects the combined standard uncertainty of the measurement process, statistically expanded to account for the level of confidence that the result is to be found within those limits.

D2. Expanded Uncertainty

The combined standard uncertainty expresses the uncertainty resulting from a number of measurements defining the measurand. By itself, the combined standard uncertainty does not reveal the possible variation in results arising from the combination of the preceding probabilities. To account for this possible variability, the term expanded uncertainty is introduced. Expanded uncertainty is obtained by multiplying the combined standard uncertainty, u_c by a factor which reflects the preceding probabilities. This factor, *k*, is termed the coverage factor.

Expanded Uncertainty**,** $U = k\, u_c$

D3 Coverage factor

D3.1 The value of the coverage factor, normally symbolised by *k*, is dependent on the desired level of confidence to be associated with the uncertainty.

For example: if we wish to be 95% confident that the uncertainty calculated is within an interval about the result, we would multiply the combine standard uncertainty by a coverage factor of 1.96, assuming normal distribution. If we wish to be 99% confident, the coverage factor would be about 3. The expanded uncertainty for being 99% confident is larger than that if we just wish to be 95% confident.

D3.2 The computation of *k* is complex and dependent on the type and interaction of the preceding probability distribution. For a combination of a large number of this probability distribution, assuming all varied distributions, the resulting distribution tends towards a normal distribution, according to Central Limit Theorem. How well this assumption is met is reflected in the term called the degree of freedom. The degree of freedom quantifies the amount of knowledge use in estimating the uncertainty.

D3.3 In type A estimate of uncertainty, the size of the sample used to estimate the uncertainty is the degree of freedom less the number of parameters so derived.

For example, if dimension is the only parameter in a series of measurement of specimen length, then the degree of freedom is n-1, where n is the number of repeated measurement of that parameter.

D3.4 In type B estimates, the selected uncertainty limits of the assumed distribution are usually chosen such that the probability of the quantity lying outside these limits is extremely small. Under this condition, the degree of freedom may be taken as infinite. That is, complete knowledge that the quantity is within the selected limits of the assumed distribution.

D3.5 The degree of freedom for the combined standard uncertainty, commonly termed the effective degree of freedom, is determined using the Welch-Satterthwaite formula:

$$\nu_{\mathrm{eff}} = \frac{u_c^4(y)}{\sum_{i=1}^{N} \frac{c_i^4\, u^4(x_i)}{\nu_i}} \quad ,$$

D3.6 If the underlying distribution for the combined estimate is normal, the *t*-distribution can be used to develop confidence limits. This is done by obtaining the coverage factor, *k*, from the *t*-distribution, based on the effective degree of freedom and the desired level of confidence. If ν_{eff} is not an integer, which will usually be the case, either interpolate or truncate ν_{eff} to the next lower integer.

D3.7 The effective degree of freedom is related to the individual degree of freedom, ν_i, and uncertainty, $c_i u_x$. If ν_i is small, simply assuming that the uncertainty of $c_i u_x$, is negligible, thus resulting in a large ν_{eff} and taking $k = 2$ may be inadequate.

Table 1. Student's *t-distribution* for various degrees of freedom for 95% confidence interval

Degrees of Freedom (*v*)	*k*	Degrees of Freedom (*v*)	*k*
1	12.7	18	2.10
2	4.30	19	2.09
3	3.18	20	2.09
4	2.78	25	2.06
5	2.57	30	2.04
6	2.45	35	2.03
7	2.37	40	2.02
8	2.31	45	2.02
9	2.26	50	2.01
10	2.23	60	2.00
11	2.20	70	2.00
12	2.18	80	1.99
13	2.16	90	1.99
14	2.15	100	1.98
15	2.13	110	1.98
16	2.12	120	1.98
17	2.11	∞	1.96

APPENDIX E

GLOSSARY OF TERMS

E1 Accuracy (of Measurement) *(VIM 3.5)*

Closeness of agreement between the result of a measurement and a true value of the measurand.

Note:

- "Accuracy" is a qualitative concept
- The term "precision" should not be used for "accuracy"

E2 Coverage Factor, k *(GUM 2.3.6)*

Numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty.

E3 Error (of Measurement) *(VIM 3.10)*

Result of a measurement minus a true value of the measurand.

Note:

- Since a true value cannot be determined, in practice a conventional true value is used.
- When it is necessary to distinguish "error" from "relative error", the former is sometimes called "absolute error of measurement". This should not be confused with "absolute value of error", which is the modulus of the error.

E4 Level of confidence *(GUM C.2.29)*

The value of the probability associated with a confidence interval or a statistical coverage interval.

E5 Measurand *(VIM 2.6)*

Particular quantity subject to measurement.

Example:

- Vapor pressure of a given sample of water at 20º C

Note:

- The specification of a measurand may require statements about quantities such as time, temperature and pressure.

E6 Measurement *(VIM 2.1)*

Set of operations having the objective of determining a value of a quantity.

E7 Type A Evaluation of Uncertainty *(GUM 2.3.2)*

Method of evaluation of uncertainty by the statistical analysis of observations

E8 Type B Evaluation of Uncertainty *(GUM 2.3.3)*

Method of evaluation of uncertainty by means other than the statistical analysis of a series of observations.

E9 Uncertainty (of a measurement) *(VIM 3.9)*

Parameter, associated with the result of a measurement that characterises the dispersion of the values that could reasonably be attributed to the measurand.

Note:

- The parameter may be, for example, a standard deviation (or a given multiple of it), or the half-width of an interval having a stated level of confidence.
- Uncertainty of measurement comprises, in general, many components. Some of these components may be evaluated from the statistical distribution of the results of series of measurements and can be characterised by experimental standard deviations. The other components, which can also be characterised by standard deviations, are evaluated from assumed probability distributions based on experience or other information.
- It is understood that the result of the measurement is the best estimate of the value of the measurand, and that all components of uncertainty, including those arising from systematic effects, such as components associated with corrections and reference standards, contribute to the dispersion.

E10 Uncertainty (Standard) *(GUM 2.3.1)*

Uncertainty of the result of a measurement expressed as a standard deviation.

E11 Uncertainty (Combined Standard) *(GUM 2.3.4)*

Standard uncertainty of the result of a measurement when that result is obtained from values of a number of other quantities, equal to the positive square root of a sum of terms, the terms being the variances or co-variances of these other quantities weighted according to how the measurement result varies with changes in these quantities.

E12 Uncertainty (Expanded) *(GUM 2.3.5)*

Quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand.

E13 Repeatability (of results of measurements) *(VIM 3.6)*

Closeness of the agreement between the results of successive measurements of the same measurand carried out under the same conditions of measurement

Notes :

- These conditions are called **repeatability conditions**
- Repeatability conditions include:
  - the same measurement procedure
  - the same observer
  - the same measuring instrument, used under the same conditions
  - the same location
  - repetition cover a short period of time
- Repeatability may be expressed quantitatively in terms of the dispersion characteristics of the results

E14 Reproducibility (of results of measurements) *(VIM 3.7)*

Closeness of the agreement between the results of measurements of the same measurand carried out under changed conditions of measurement

Notes

- A valid statement of reproducibility requires specification of the conditions changed
- The changed conditions may include:
  - principle of measurement
  - method of measurement
  - observer
  - measuring instrument
  - reference standard
  - location
  - conditions of use
  - time
- Reproducibility may be expressed quantitatively in terms of the dispersion characteristics of the results
- Results are here usually understood to be corrected results

Note:

* GUM – Guide to the Expression of Uncertainty in Measurement (1995), Published by ISO
* VIM – International Vocabulary of basic and general terms in Metrology (1993), Published by ISO