Measurement uncertainty analysis of CMM with ISO GUM

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Abstract

This paper states the measurement uncertainty estimation of the Coordinate Measurement Machine (CMM) in accordance with the *Guide to the Expression of Uncertainty in Measurement*, abbreviated as ISO *GUM* [1]. The analysis result shows that the measurement uncertainties mainly come from the calibration of CMM and temperature. The calibration uncertainty of the CMM plays an important role in its measurement uncertainty. If the calibration uncertainty could be reduced, it will help decrease the measurement uncertainty of the CMM. Temperature is really a great influential factor in dimensional measurement, especially in precision measurement. Therefore, sufficient thermal equilibrium should be achieved before measurement. The environmental temperature of the laboratory should be controlled at a specific condition to ensure that the CMM can work at its specifications. Finally, the measurement uncertainty has been evaluated by using a set of measured data. For a measurement range of 0 mm to 400 mm, the estimated expanded uncertainty is 3.4 μm with a coverage factor of 1.98 at a confidence level of approximately 95 %. Analysis shows that measurement uncertainty can be reduced, if we use a high precision instrument such as laser interferometer.

Introduction

CMM is one of the best dimensional and geometrical measurement instruments. The manufacturer always finds ways to improve the performance. There are specifications and researches focusing on CMM performance evaluation, such as ANSI/ ASTM B89.4.1 [2], VDI/VDE 2617 [3], and ISO 10360 [4]. But performance improvement does not equivalence to the improvement in reliability of the measurement. Using an instrument with good performance does not imply to obtain good measurement results. It is because that the reliability of measurement results is not only attributed to the instrument, but also the environment, operator, and method are involved. It is better to use the term of measurement uncertainty to characterize the reliability of measurement results.

Many researches were investigated dealing with the uncertainty of CMM measurement results in recent years. Weckenmann used factorial analysis and fishbone diagrams to evaluate uncertainty instead of the *GUM* method [5]. Yan applied the coordinates transformation to the CMM for the calculation of measurement results. If the uncertainties of coordinates transformation of each stage were obtained, the total uncertainty can be obtained [6]. A DOE method was used to assess the measurement uncertainty of CMM in Piratelli-Filho's research [7]. Some error sources were considered. The location and the length of test item were selected to the factorial design. Three levels were taken into cross comparison. ANOVA method was used to decide which one is significant. Finally, the results were combined as the total uncertainty. Abbe believed that a well-calibrated CMM performs good measurement results [8]. A traditional calibration was treated to each components of the CMM separately in Abbe's work. The errors of coordinates transformation were analysed and calculated. The error components were then combined to get the error model, but the combination error itself was not considered. The environmental and operational errors may be ignored, too.

CMMs are the most important and well-developed measurement instruments. They are widely used to measure the product's dimensions in the advanced semi-conductor industry. Therefore, the products' quality can be influenced by the reliability and quality of the measurement results. A model was developed in accordance with ISO *GUM* to fit the CMM measurement in this paper. Error sources were found to estimate their standard uncertainties separately. The combined standard uncertainty was then calculated. The principles of most dimensional instruments are similar to that of the CMM. This paper can be referred as a guide to evaluate the measurement uncertainties of such instruments.

Measurement uncertainty analysis

According to ISO *GUM*, the measurement should be modelled to give a mathematical equation. The measurement principle of CMM is shown in figure 1.

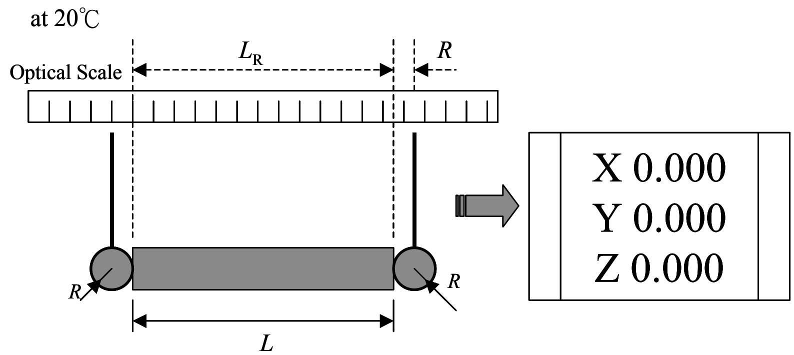

Figure 1 Schematic measurement principle of CMM

Temperature compensations were taken into consideration to correct the length measured at 20 °C. The mathematic model can be expressed as in Eq. (1).

$$L(1+\alpha\theta) = L_R(1+\alpha_g\theta_g) + 2R(\alpha_g\theta_g - \alpha_p\theta_p) + \Delta L \tag{1}$$

or $$L = L_R(1+\alpha_g\theta_g - \alpha\theta) + 2R(\alpha_g\theta_g - \alpha_p\theta_p) + \Delta L(1-\alpha\theta) \tag{2}$$

where L : measurand, the length of the test item at 20 °C

L_R : reading of the CMM at 20 °C

R : radius of the probe at 20 °C

ΔL : axial calibrated value of CMM

α, α_g, α_p : thermal expansion coefficients of test item, optical scale, and probe, respectively.

θ, θ_g, θ_p : temperature deviations of test item, optical scale, and probe from 20 °C, respectively.

Since the variables θ, θ_g, and θ_p are dependent to each other, the following transformations lead them to be independent.

$\delta\theta_g = \theta_g - \theta$: Temperature difference between optical scale and test item

$\delta\alpha_g = \alpha_g - \alpha$: Thermal expansion coefficient difference between optical scale and test item

$\delta\theta_p = \theta_p - \theta$: Temperature difference between probe and test item

$\delta\alpha_p = \alpha_p - \alpha$: Thermal expansion coefficient difference between probe and test item

Thus, we get $\theta_g = \delta\theta_g + \theta$, $\alpha_g = \delta\alpha_g + \alpha$, $\theta_p = \delta\theta_p + \theta$, and $\alpha_p = \delta\alpha_p + \alpha$. Substitute them into Eq. (2) and ignore the higher order terms. A simplified equation can be obtained as in Eq. (3).

$$L = L_R(1+\theta\delta\alpha_g + \alpha\delta\theta_g) + 2R[\alpha(\delta\theta_g - \delta\theta_p) + \theta(\delta\alpha_g - \delta\alpha_p)] + \Delta L(1-\alpha\theta) \tag{3}$$

The temperature variables in Eq. (3) are θ, $\delta\theta_g$, and $\delta\theta_p$, which are independent. Since the CMM origin must be set with standard ball before measurement, the origin setting will affect the measurement results. Thus, a term ε will be added to Eq. (3). ε is so-called reset error that mentioned in ISO *GUM* 3.3.2 for non-complete definition of measurand. The value of ε can be deemed to be zero, but not its standard uncertainty $u(\varepsilon)$. Eq. (3) can be modified as Eq. (4).

$$L = L_R(1+\theta\delta\alpha_g + \alpha\delta\theta_g) + 2R[\alpha(\delta\theta_g - \delta\theta_p) + \theta(\delta\alpha_g - \delta\alpha_p)] + \Delta L(1-\alpha\theta) + \varepsilon \tag{4}$$

or $L = f\left(L_{\mathrm{R}}, R, \Delta L, \alpha, \delta\alpha_{\mathrm{g}}, \delta\alpha_{\mathrm{p}}, \theta, \delta\theta_{\mathrm{g}}, \delta\theta_{\mathrm{p}}, \varepsilon\right)$

The combined standard uncertainty $u_{\mathrm{c}}(L)$ can be obtained by Eq. (5) according to the uncertainty propagation law.

$$\begin{aligned} u_{\mathrm{c}}^{2}(L) = & \left(\frac{\partial L}{\partial L_{\mathrm{R}}}\right)^{2} u^{2}(L_{\mathrm{R}}) + \left(\frac{\partial L}{\partial R}\right)^{2} u^{2}(R) + \left(\frac{\partial L}{\partial \Delta L}\right)^{2} u^{2}(\Delta L) + \left(\frac{\partial L}{\partial \alpha}\right)^{2} u^{2}(\alpha) + \left(\frac{\partial L}{\partial \delta\alpha_{\mathrm{g}}}\right)^{2} u^{2}(\delta\alpha_{\mathrm{g}}) \\ & + \left(\frac{\partial L}{\partial \delta\alpha_{\mathrm{p}}}\right)^{2} u^{2}(\delta\alpha_{\mathrm{p}}) + \left(\frac{\partial L}{\partial \theta}\right)^{2} u^{2}(\theta) + \left(\frac{\partial L}{\partial \delta\theta_{\mathrm{g}}}\right)^{2} u^{2}(\delta\theta_{\mathrm{g}}) + \left(\frac{\partial L}{\partial \delta\theta_{\mathrm{p}}}\right)^{2} u^{2}(\delta\theta_{\mathrm{p}}) + \left(\frac{\partial L}{\partial \varepsilon}\right)^{2} u^{2}(\varepsilon) \end{aligned} \tag{5}$$

where $\dfrac{\partial L}{\partial x_i}$ is the sensitivity coefficient which is derived as below.

$$\frac{\partial L}{\partial L_{\mathrm{R}}} = 1 + \theta\delta\alpha_{\mathrm{g}} + \alpha\delta\theta_{\mathrm{g}}; \qquad \frac{\partial L}{\partial R} = 2\alpha\left(\delta\theta_{\mathrm{g}} - \delta\theta_{\mathrm{p}}\right) + \theta\left(\delta\alpha_{\mathrm{g}} - \delta\alpha_{\mathrm{p}}\right);$$

$$\frac{\partial L}{\partial \Delta L} = 1 - \alpha\theta; \qquad \frac{\partial L}{\partial \theta} = L_R\delta\alpha_g + 2R(\delta\alpha_g - \delta\alpha_p) - \alpha\Delta L;$$

$$\frac{\partial L}{\partial \alpha} = L_{\mathrm{R}}\delta\theta_{\mathrm{g}} - \Delta L\theta; \qquad \frac{\partial L}{\partial \delta\theta_{\mathrm{g}}} = \left(L_{\mathrm{R}} + 2R\right)\alpha;$$

$$\frac{\partial L}{\partial \delta\alpha_{\mathrm{g}}} = \left(L_{\mathrm{R}} + 2R\right)\theta; \qquad \frac{\partial L}{\partial \delta\theta_{\mathrm{p}}} = 2R\alpha;$$

$$\frac{\partial L}{\partial \delta\alpha_{\mathrm{p}}} = -2R\theta; \qquad \frac{\partial L}{\partial \varepsilon} = 1;$$

By substituting the values of each parameter, such that $L_{\mathrm{R}} = 0.04$ m, $\Delta L = 1.6\times10^{-6}$ m, $R = 0.003$ m, $\alpha = 11.5\times10^{-6}$ °C^{-1}, $\delta\alpha_{\mathrm{g}} = -3.5\times10^{-6}$ °C^{-1}, $\delta\alpha_{\mathrm{p}} = -6.5\times10^{-6}$ °C^{-1}, $\theta = 1$ °C, $\delta\theta_{\mathrm{p}} = 0.2$ °C, and $\delta\theta_{\mathrm{g}} = 0.2$ °C, into the above equations, we can obtain the uncertainty budget as tabulated in Table 1. The uncertainty magnitudes of each error source can be compared and the contribution of each uncertainty component to the expanded uncertainty can be analysed.

Conclusions

An evaluation model of CMM measurement uncertainty is established according to ISO *GUM*. For a measurement range of 0 mm to 400 mm, the estimated expanded uncertainty is 3.4 μm with a coverage factor of 1.98 at a confidence level of 95 %. From Table 1, it was found that the uncertainty is mainly come from two sources. They are the traceability of the CMM and the temperature influence. For the traceability, the CMM shall be calibrated periodically to ensure its accuracy. Then, accurate measurement results can be obtained. By the law of propagation of uncertainty, the measurement uncertainty of the CMM calibration will contribute to the uncertainty of the measurement result taken by the CMM. Since the CMM is calibrated by traditional method, which uses gauges resulting a bigger calibration uncertainty. If the CMM is calibration by using laser interferometer, the calibration uncertainty will be reduced. Thus, uncertainty due to traceability can be reduced. Temperature is a very important factor to dimensional measurement, especially to precision measurement. Thus, temperature influence shall be taken into consideration on the uncertainty estimation. The environmental temperature shall be well controlled inside the laboratory. The test item shall be placed on the CMM for an enough time period to achieve thermal equilibrium before taking the measurement. In this study, the uncertainty due to temperature is evaluated by type B estimation. The temperature parameters are substituted by bounds of temperature variation for conservative estimation, which gives a bigger uncertainty. If temperature sensors are installed on the CMM's axes and the test item, the temperatures of the CMM and test item can be instantly captured for temperature compensation. Thus, the measurement uncertainty will be reduced.

References

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Table 1 Uncertainty budget

Error source x_i	Type	Standard uncertainty $u(x_i)$	Sensitivity coefficient $\frac{\partial L}{\partial x_i}$	Uncertainty component $\left\lvert\frac{\partial L}{\partial x_i}\right\rvert u(x_i)$	DOF $\nu(x_i)$
Reading of CMM (L_R)		0.143 μm	1	0.143 μm	31.3
Random effect	A	0.10 μm			14
Imperfect etching scale	B	0.102 μm			12.5
Traceability (ΔL)	B	1.162 μm	1	1.162 μm	50
Probe radius (R)		0.175 μm	3.0×10^{-6}	5.3×10^{-7} μm	547
Calibration of std. ball	B	0.16 μm			∞
Calibration of probe	A	0.07 μm			14
Thermal expansion coefficient of test item (α)	B	0.866×10^{-6} °C^{-1}	0.08 m °C	0.069 μm	50
Difference of thermal expansion coefficient between optical scale and test item ($\delta\alpha_g$)	B	0.8165×10^{-6} °C^{-1}	0.406 m °C	0.332 μm	12.5
Difference of thermal expansion coefficient between probe and test item ($\delta\alpha_p$)	B	1.0206×10^{-6} °C^{-1}	0.006 m °C	0.006 μm	12.5
Temperature deviation of test item from 20 °C (θ)	B	0.707 °C	-1.382×10^{-6} m °C^{-1}	0.977 μm	50
Temp. difference between optical scale and test item ($\delta\theta_g$)	B	0.1414 °C	4.669×10^{-6} m °C^{-1}	0.660 μm	50
Temperature difference between probe and test item ($\delta\theta_p$)	B	0.1414 °C	6.9×10^{-8} m °C^{-1}	0.0098 μm	50
Reset error (ε)	A	0.289 μm	1	0.289 μm	12.5
Combined standard uncertainty $u_c(L)$ = 1.72 μm Effective degrees of freedom ν_{eff} = 146 Coverage factor k = 1.98 Expanded uncertainty $U = k\cdot u_c(L)$ = 3.4 μm (confidence level of 95 %)					