Estimation of uncertainties in ICP-MS analysis: a practical methodology

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Abstract

The measurement uncertainty associated with the determination of $^{60}$Ni in aqueous samples by ICP-MS has been calculated using a cause-and-effect approach. A cause-and-effect diagram was constructed to aid in the identification of the sources of uncertainty associated with the method. The uncertainty estimate was calculated from a combination of existing quality control data and specially planned experiments. The uncertainty budget was based initially on precision data, followed by separate evaluation of the method bias and the effect of parameters not sufficiently covered by these estimates. The construction of the cause-and-effect diagram, its reconciliation with existing data and the estimation of the individual components of the uncertainty budget are described in detail. The expanded uncertainties for three different nickel concentrations (3, 10, 35 ng g$^{-1}$) were calculated as 1.1, 1.5 and 5.3 ng g$^{-1}$, respectively. These were calculated using a coverage factor of two approximating to a 95% level of confidence. The dominant contributions to the uncertainty budget were method precision, instrument drift and bias measured as method recovery. The uncertainties associated with the concentration of the working standard and sample dilution were found to be insignificant. © 1999 LGC (Teddington Ltd). Published by Elsevier Science B.V. All rights reserved.

Keywords: Uncertainty budget; Cause-and-effect analysis; ICP-MS; Nickel; Water

1. Introduction

It is now widely recognised that the evaluation of the uncertainty associated with a result is an essential part of any quantitative analysis [1]. The statement of an analytical result is, therefore, not considered complete without an indication of the uncertainty associated with it. With increasing accreditation and quality control (QC) procedures being implemented in all facets of analytical chemistry, measurement uncertainty is beginning to have a higher profile and, indeed, analytical methods accredited in accordance with ISO Guide 25 [2] require an estimate of the associated measurement uncertainty. This wider use of properly calculated uncertainty estimates will ultimately allow improved intercomparability of analytical results. However, in the short term, there is a need for the education, not only of the final customer in being able to interpret the uncertainty estimated, but also for the analyst in being able to make an informed estimation of the uncertainty of a particular technique.
Recently, the International Standards Organisation (ISO) *Guide to Expression of Uncertainty in Measurement* [3] (GUM) has been interpreted for analytical chemistry applications by the Eurachem organisation [4]. The most important characteristic of a properly calculated uncertainty budget, compared to other measures of method performance, is that it encompasses both random and systematic effects to give a single value [1,3,4]. The approach described in the GUM and, subsequently, in the Eurachem guide, involves the identification of all the possible sources of uncertainty for the method; the estimation of their magnitude from either published or experimental data; and the combination of these individual uncertainties to give standard and expanded estimates. Although some applications of this approach to analytical chemistry have been published [5,6], relatively few uncertainty budgets have been published. The GUM principles are significantly different from the methods currently used in analytical chemistry for estimating uncertainty [7–9] which generally make use of ‘whole method’ performance parameters such as precision (repeatability, reproducibility and other precision measures) and recovery (where recovery is defined as the ratio of the observed to the expected result). The latter may be based on, for example, the analysis of reference materials or spiked samples. We have developed a strategy for reconciling the requirements of formal measurement uncertainty (i.e. GUM) principles with the data commonly available in analytical laboratories from method validation studies and QC protocols [10–12]. The approach involves a detailed analysis of the factors influencing the analytical result using cause-and-effect analysis. This results in a structured list of the possible sources of uncertainty associated with the method. The list is then simplified and reconciled with existing experimental and other data.

In this paper, we describe the application of this strategy to an Inductively Coupled Plasma-Mass Spectrometry (ICP-MS) technique for the quantification of $^{60}$Ni in water samples. Fully worked equations and the cause-and-effect diagram are explained in a manner for the practical analyst. Although nickel is considered here, similar calculations can be undertaken for other routinely analysed elements for which QC and/or method validation data are available.

### 2. Experimental

#### 2.1. Instrumentation

All determinations were carried out using a Perkin–Elmer ELAN 5000A ICP-MS (Perkin–Elmer, Beaconsfield, UK). The operating conditions for the ICP-MS system are given in Table 1. Before all analyses, the instrument was checked for mass response curve, resolution, and oxide and doubly charged ions according to the instrument manufacturer’s instructions [13].

#### 2.2. Reagents and standard solutions

All solutions were prepared with high-purity deionized water (18 MΩ, Elgar, High Wycombe, UK). Nickel standards and rhodium internal standard solutions were prepared daily from 1000 μg ml$^{-1}$ stock solutions (Alfa, Johnson Matthey, Royston, Herts., UK). The certified reference material used in the recovery study was SRM 3136, nickel in 10% nitric acid, supplied by the National Institute of Standards and Technology (NIST), USA. All acid solutions used for dilution were prepared from ultrapure Ultrex II grade acid stock solutions (J.T. Baker, Phillipsburg, NJ, USA). All solutions and standards were prepared gravimetrically.

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**Table 1**

ICP-MS operating conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ICP</strong></td>
<td></td>
</tr>
<tr>
<td>Power</td>
<td>1000 W</td>
</tr>
<tr>
<td>Plasma gas</td>
<td>15.0 l min$^{-1}$</td>
</tr>
<tr>
<td>Auxiliary gas</td>
<td>0.8 l min$^{-1}$</td>
</tr>
<tr>
<td>Nebuliser gas</td>
<td>0.90 l min$^{-1}$</td>
</tr>
<tr>
<td>Cones</td>
<td>Pt</td>
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<tr>
<td>Lenses</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>44 (switch settings)</td>
</tr>
<tr>
<td>B</td>
<td>48</td>
</tr>
<tr>
<td>S</td>
<td>45</td>
</tr>
<tr>
<td>E</td>
<td>25</td>
</tr>
<tr>
<td><strong>Data acquisition</strong></td>
<td></td>
</tr>
<tr>
<td>Masses</td>
<td>$^{60}$Ni, $^{103}$Rh</td>
</tr>
<tr>
<td>Dwell time</td>
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</tr>
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<tr>
<td>Readings/replicates</td>
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</tr>
<tr>
<td>Number of replicates</td>
<td>6</td>
</tr>
<tr>
<td>Points across peak</td>
<td>1</td>
</tr>
<tr>
<td>Resolution</td>
<td>normal</td>
</tr>
</tbody>
</table>
2.3. Brief description of the ICP-MS method

The ICP-MS method described here has been developed for the multi-element analysis of water samples. The $^{60}$Ni isotope was chosen as a typical interference free isotope for this type of matrix, thus simplifying the process of formulating the uncertainty budget. Typically, calibration standards are prepared from a 10 μg g$^{-1}$ working standard solution which, in turn, has been prepared from a 1000 μg ml$^{-1}$ stock solution of nickel. A solution of 5–10 ng g$^{-1}$ rhodium is added on-line via a simple T-piece and used as the internal standard. A typical analytical run would consist of standards, blanks, samples and standards again. A check or drift standard is run every six samples with any subsequent drift corrections being performed off-line.

2.4. Identification of sources of uncertainty

2.4.1. Construction of the cause-and-effect diagram

The sources of uncertainty for the method were identified by constructing a cause-and-effect diagram [14]. The application of cause-and-effect analysis to analytical methods is described in detail elsewhere [10,11]. The ‘effect’, represented by the main horizontal branch in the diagram (see Fig. 1), is the result of the analysis, i.e. the concentration of nickel in ng g$^{-1}$. The main ‘cause’ branches represent the main parameters controlling the result. These are shown in Eq. (1).

$$C_{Ni} = C' \times D \times \frac{1}{R}$$

(1)

where $C'$ is the concentration of the sample solution as read from the calibration curve, $D$ the dilution factor applied to the sample, and $R$ the recovery factor.$^1$

The uncertainties associated with these parameters will contribute to the overall uncertainty in the final result. The uncertainties of these parameters will, in turn, have contributions from other stages in the method. These are, in turn, represented by branches on the cause-and-effect diagram. The cause-and-effect diagram is thus expanded by continuing this process until the effects become sufficiently remote, i.e. until effects on the result are negligible. The cause-and-effect diagram for this procedure is presented in Fig. 1. Fortunately, as will be demonstrated later, it is not usually necessary to evaluate all of these contributions individually to obtain the overall uncertainty of the method.

2.5. Simplification of the cause-and-effect diagram

Once a cause-and-effect diagram has been constructed it generally requires simplification to resolve any duplication of components. This resulting diagram can then be used to identify the components for which uncertainty estimates are required, and reconciled with existing data. The simplification of the cause-and-effect diagram is discussed below, taking each major branch in isolation.

2.5.1. Recovery, $R$

The overall recovery, $R$, for a particular sample can usefully be considered as comprising two components, $R_m$ and $R_s$, where

$$R = R_m \times R_s,$$

$R_m$ being an estimate of the recovery for the entire procedure, including preparation of calibration standards and any dilution of the sample (a ‘method recovery’). $R_m$ is measured (ideally) on a suitable reference sample, or perhaps as a mean recovery over many materials. This value has an uncertainty associated with the reference value used and with the variability of the particular measurement of the recovery on that material. However, as $R_m$ represents a test of bias against a particular reference value, it is also necessary to consider differences between it and the recovery for ‘real’ samples. $R_s$ represents this difference between the reference and a particular sample. While expected to be identically 1.0, the effect of variations between different materials appears as an uncertainty in $R_s$. $u(R_s)$ thus describes the variation in recovery between the different sample matrices and different analyte levels permitted by the method scope. The uncertainty associated with $R_s$ is $u(R_s)$, accordingly has contributions from $u(R_m)$ and $u(R_s)$.

2.5.2. Dilution factor, $D$

In the ICP-MS method employed, all samples are diluted by a factor of 10 with 1% nitric acid on w/w basis. The two contributions to the uncertainty asso-
Fig. 1. Cause-and-effect diagram for the analysis of Ni by ICP-MS.
2.5.3. Precision, $P$

The precision branch collects terms which contribute to the random variability of the entire method. Estimates of precision are available from a number of sources, such as QC data and replicate analysis of samples. In general, if an operation was repeated during the period in which the precision data were obtained, the run-to-run variability associated with that operation will be included in the overall precision estimate and a separate estimate is not required.

2.5.4. Concentration, $C_0$

As shown in Fig. 1, there are three major contributions to the uncertainty associated with the concentration of the analyte, $C_0$. These are represented by the branches $f(I_{\text{ref}i}, C_{\text{ref}i}, I_{\text{sample}})$, $C_{\text{stock}}(w/w)$ and $C_{\text{dil}}(w/w)$. $f(I_{\text{ref}i}, C_{\text{ref}i}, I_{\text{sample}})$ represents the application of the calibration function, obtained from a series of reference concentrations $C_{\text{ref}i}$ and observed intensities $I_{\text{ref}i}$, to the observed sample intensity, $I_{\text{sample}}$, in order to obtain the interpolated concentration figure for the solution.

For this particular ICP-MS method, the concentration of the 1000 $\mu$g ml$^{-1}$ stock solution has to be calculated in terms of weight-by-weight. This is achieved using the density data provided by the supplier of the stock solution. The second source of uncertainty that contributes to the uncertainty in $C_0$ is the concentration of the dilute working standard, $C_{\text{dil}}$, from which the calibration solutions are prepared. This solution is prepared by diluting the stock solution on a weight-by-weight basis. As for the dilution of the sample, the balance zero-bias terms will cancel. The only uncertainties associated with the concentration of the dilute working standard (apart from run-to-run variations in preparing the standard) which need to be considered are the uncertainties about $C_{\text{stock}}(w/w)$ and associated balance non-linearity terms.

Finally, the uncertainty associated with the calibration function $f(I_{\text{ref}i}, C_{\text{ref}i}, I_{\text{sample}})$ needs to be considered. The ratio counts for the standards and sample ($I_{\text{ref}i}$ and $I_{\text{sample}}$, respectively) will be affected by the instrument performance. The instrument is calibrated for each batch of samples with a fresh set of calibration standards. Therefore, systematic effects relating to the instrument performance should cancel out as they will be the same for both, the calibration standards and the sample solutions. In addition, during a batch of analyses one of the calibration standards is analysed periodically to ensure that the instrument response has not drifted significantly. If a drift of $>10\%$ is observed, then the instrument is recalibrated and the samples reanalysed. A term representing the uncertainty due to this maximum permitted drift also needs to be included in the budget. The contribution to the overall uncertainty from the run-to-run variability of the instrument performance should be included in an estimate of the overall precision of the method. It is, therefore, not necessary to obtain individual uncertainty estimates for the components feeding into both, the $I_{\text{ref}i}$ and $I_{\text{sample}}$ branches.

The above considerations lead to a simplified cause-and-effect diagram. The parameters removed in the simplification process are represented by italic text and arrows with broken lines in Fig. 1. The simplified diagram can be used as a check list to ensure that all the contributions to the measurement uncertainty are being adequately covered.

3. Results and discussion

3.1. Reconciliation of simplified cause-and-effect diagram with existing data

Simplification of the cause-and-effect diagram is followed by a reconciliation stage. The sources of uncertainty identified in the cause-and-effect diagram are compared with existing experimental and other data to determine which are covered and which require further evaluation. The following information was available for this method:

- QC data—replicate analysis at the beginning of each batch of a stable sample originating from a proficiency testing (PT) scheme
• Precision data for the balance used to prepare the samples and standards
• Estimate, given by the supplier, of the uncertainty associated with the concentration of the 1000 μg ml\(^{-1}\) nickel solution used to prepare the calibration standards
• Information from the supplier on the density of the 1000 μg ml\(^{-1}\) nickel stock solution

The QC data were obtained over a number of months and can, therefore, be used to obtain an estimate of the long-term precision for the method. However, the data do not completely cover all sources of variability in the method. The QC sample is not diluted prior to analysis so the run-to-run variability associated with the dilution of samples requires separate evaluation. Although a new set of calibration solutions is prepared for each batch of analysis, the dilute working standard from which they are prepared remains stable over several months, so the period during which the QC data were obtained did not include the preparation of a new dilute working standard. The precision associated with this operation, therefore, also requires separate evaluation. The assumption about the stability of the standard will contribute to the measurement uncertainty as its concentration may change during this period. However, as the data from the QC sample were obtained using calibration standards prepared from the same dilute working standard, any variation in results due to changes in the concentration of this solution will already be included in the overall precision estimate.

The QC sample originates from a PT scheme. The assigned value for the sample is based on the consensus of the results returned by laboratories participating in the scheme, and is not therefore traceable. To obtain a reliable estimate of the mean recovery for the method, \(R_m\), a traceable value, preferably from a certified reference material, is required. Other information lacking was an estimate of the variation in recovery with analyte concentration and change of matrix, (represented by \(u(R_m)\)). Experiments to obtain these data are described below.

Finally, as a new set of standards is prepared for each batch of analyses, the run-to-run variability associated with the calibration of the instrument will be included in the overall precision estimate. However, unless the QC sample is placed at random within each batch, the uncertainty due to instrumental drift during a batch of analyses will not appear in the dispersion of QC results, and therefore needs to be considered. A drift of 10% is permitted in the method before the instrument is recalibrated. Although the within-run precision was normally 1–2% for single isotope monitoring, with the multi-element program that the \(^{60}\)Ni formed part of, up to 12–15 isotopes are monitored. Thus, to accommodate drift of up to 2 × the within-run precision (as stated in the method), this value is normally increased to 10%. The result for any particular sample could, therefore, have drifted by up to 10% compared to a reading taken at the time of calibration. This is taken as the basis for the estimate of the uncertainty associated with within-batch drift (see later discussion).

3.2. Determination of the magnitude of uncertainty components

3.2.1. Recovery

The estimate of \(R_m\) and \(u(R_m)\) (method recovery and method recovery uncertainty) was obtained from standards prepared by serial dilution of a NIST certified reference solution (SRM 3136) with 1% nitric acid. The concentration of the solution, \(C_{SRM(w/w)}\), is certified as 9.95 ± 0.03 mg ml\(^{-1}\). The uncertainty is quoted at the 95% confidence level. The density of the solution, \(\rho_{SRM}\), is quoted by the supplier as 1.112 ± 0.002 g cm\(^{-3}\). The concentration of nickel in the solution calculated on a weight/weight basis, \(C_{SRM(w/w)}\), is 8947.8 μg g\(^{-1}\). The uncertainty, \(u(C_{SRM(w/w)})\), is obtained by combining the uncertainty in the certified value, \(u(C_{SRM(w/w)})\), with the uncertainty in the density as follows [3,4]:

\[
\frac{u(C_{SRM(w/w)})}{C_{SRM(w/w)}} = \sqrt{\left(\frac{u(C_{SRM(w/w)})}{C_{SRM(w/w)}}\right)^2 + \left(\frac{u(\rho_{SRM})}{\rho_{SRM}}\right)^2}
\]

(2)

Therefore:

\[
u(C_{SRM(w/w)}) = 8947.8 \times \sqrt{\left(\frac{0.015}{9.95}\right)^2 + \left(\frac{0.00115}{1.112}\right)^2}
\]

\[
\equiv 16.4 \mu g \ g^{-1}
\]

(3)

\(^2\)The uncertainty quoted for the density of the solution is assumed to be a rectangular distribution and is, therefore, converted to a standard uncertainty by dividing by \(\sqrt{3}\).
Two solutions were prepared by serial dilution of the certified solution. Solution A had a nickel concentration of 9.58 ng g$^{-1}$, whilst solution B had a concentration of 51.12 ng g$^{-1}$. The uncertainty associated with the concentration of the certified solution calculated in Eq. (3) equates to a relative standard uncertainty of 0.18%. Previous studies [4,12] have shown that the uncertainties associated with weighing operations will be small compared to this. It is, therefore, reasonable to assume that the uncertainties associated with the concentration of the certified solution. The relevant standard uncertainties are therefore 0.0175 ng g$^{-1}$ for solution A and 0.0934 ng g$^{-1}$ for solution B. Each solution was analysed in replicate in a single analysis run. For solution A, the mean was 9.37 ng g$^{-1}$ with a standard deviation of 0.061 ng g$^{-1}$ (n = 6). For solution B, the mean was 50.20 ng g$^{-1}$ with a standard deviation of 0.341 ng g$^{-1}$ (n = 4). $R_m$ for each solution is calculated using Eq. (4):

$$R_m = \frac{c_{\text{obs}}}{C_{\text{RM}}}$$  \hspace{1cm} (4)

where $c_{\text{obs}}$ is the mean of the results obtained from the replicate analyses of the solution and $C_{\text{RM}}$ is the concentration of the solution. For solution A, $R_{m(A)} = 0.978$, and for solution B, $R_{m(B)} = 0.982$.

The uncertainty associated with $R_m$, $u(R_m)$, is obtained by combining the uncertainty in the reference value, $u(C_{\text{RM}})$ with the uncertainty in the mean of the observations [15]:

$$u(R_m) = R_m \times \sqrt{\left(\frac{u(C_{\text{RM}})}{C_{\text{RM}}}\right)^2 + \frac{s_{\text{obs}}^2}{n \times s_{\text{obs}}^2}}$$  \hspace{1cm} (5)

where $u(C_{\text{RM}})$ is the standard uncertainty associated with the certified concentration of the solution, $s_{\text{obs}}$ the standard deviation obtained from the replicate analyses of the solution, and n the number of replicates.

For solution A, $u(R_{m(A)}) = 0.00316$, and for solution B, $u(R_{m(B)}) = 0.00379$. It can be seen from the estimates of $R_{m(A)}$ and $R_{m(B)}$ and their associated uncertainties that the recoveries at the two concentration levels are very similar. It is, therefore, possible to pool the two estimates to give a single estimate of $R_{m(\text{pool})}$ and its uncertainty, $u(R_{m(\text{pool})})$.

$$R_{m(\text{pool})} = \frac{(0.978 + 0.982)}{2} = 0.98$$

$$u(R_{m(\text{pool})}) = \sqrt{\frac{0.00316^2 + 0.00379^2}{2}} = 0.0103$$  \hspace{1cm} (7)

The contribution of $R_{m(\text{pool})}$ and $u(R_{m(\text{pool})})$ to the overall uncertainty budget depends on whether $R_{m(\text{pool})}$, taking into account $u(R_{m(\text{pool})})$, is significantly different from 1 [15]. To determine this, the ratio $(1 - R_{m(\text{pool})})/u(R_{m(\text{pool})})$ is compared with the coverage factor $k$, in this case 2 (representing a confidence level of 95%), which will be used to calculate the expanded uncertainty of the overall method. This computes to a value of 8.1. A value >2 indicates that the recovery is significantly different from 1. However, in the routine application of the method, the difference is not considered to be of practical significance and no correction to the final result is applied. In such cases, the uncertainty associated with the method recovery must be increased to take account of this uncorrected bias. The increased uncertainty, $u(R_m')$, is given by:

$$u(R_m') = \sqrt{\left(\frac{1 - R_{m(\text{pool})}}{k}\right)^2 + u(R_{m(\text{pool})})^2}$$  \hspace{1cm} (8)

where $k$ is the coverage factor that will be used in the calculation of the expanded uncertainty. In this case, $u(R_m') = 0.0103$.

3.2.2. Sample recovery

In this case, the main source of variation between samples will be the concentration of nickel present. An estimate of the uncertainty associated with variations in recovery with analyte level is therefore required. This was investigated by diluting the QC sample to two different concentrations and analysing the solutions in five different batches. The results are summarised in Table 2. Analysis of variance at the

| Table 2 | Results from the analysis of $^{60}$Ni QC samples |
|-----|------------------|------------------|------------------|
| QC | QC dilution 1 | QC dilution 2 |
| Concentration (ng g$^{-1}$) | 32.42 | 10.36 | 3.14 |
| Mean | 34.71 | 11.28 | 3.43 |
| Standard deviation | 1.48 | 0.672 | 0.614 |
| Relative standard deviation | 0.0426 | 0.0596 | 0.179 |
| n | 16 | 5 | 5 |
95% confidence level, gave a negative between group variances component, leading to a maximum likelihood estimate of zero for the between group variance [16]. In other words, the difference in analyte level does not contribute significantly to the variability in the recovery. This is confirmed by the fact that the estimates of \( R_m \) obtained from solutions with concentrations of \( \approx 10 \text{ ng g}^{-1} \) and \( 50 \text{ ng g}^{-1} \) were similar.

### 3.2.3. Precision

Comparison of the relative standard deviations of the results obtained at the 10 ng g\(^{-1}\) and 35 ng g\(^{-1}\) levels (see Table 2) showed no significant difference (applying a two-tailed F-test at the 95% confidence level). The estimates were, therefore, pooled to give a single estimate of the uncertainty due to the overall method precision for samples in this concentration range. The pooled relative standard deviation was calculated as 0.0467. The relative standard deviation of the results obtained for the 3 ng g\(^{-1}\) solution was clearly greater than those obtained at the other concentration levels. Therefore, a separate estimate of the uncertainty associated with method precision is required for the lower levels. The value 0.179 was taken as the relative standard deviation of the results at 3 ng g\(^{-1}\).

### 3.2.4. Dilution factor

As mentioned above, it is a standard practice in this ICP-MS method to dilute all samples by a factor of 10 with 1% H\(\text{NO}_3\). The dilution factor, \( D \), is given by:

\[
D = \frac{W_f}{W_s} \tag{9}
\]

where \( W_s \) is the weight of sample taken and \( W_f \) the final weight of the sample after dilution. Replicate weighings of 1 and 10 g check weights on the analytical balance used for this method had standard deviations of 0.000060 and 0.00012 g, respectively. These values can therefore be used as estimates of the uncertainty associated with the precision of the balance. The uncertainty associated with balance linearity has not been considered here as previous studies in this laboratory [12] and elsewhere [4] have shown this to be an insignificant contribution to the overall uncertainty. The uncertainty associated with the dilution factor, \( u(D) \), is therefore:

\[
u(D) = 10 \times \sqrt{\left(\frac{0.00006}{1}\right)^2 + \left(\frac{0.00012}{10}\right)^2} \equiv 0.00061 \text{ g} \tag{10}
\]

### 3.2.5. Concentration of dilute working standard

The dilute working standard (nominal concentration 10 \( \mu \text{g g}^{-1} \)) is prepared by diluting 1 g of a stock solution (nominal concentration 1000 \( \mu \text{g ml}^{-1} \)) to 100 g. The concentration of the stock solution is converted to \( \mu \text{g g}^{-1} \) by dividing the concentration in \( \mu \text{g ml}^{-1} \) by the supplier’s stated density for the stock solution. The stock solution used in these experiments was quoted as having a concentration of 1000 ± 3 \( \mu \text{g ml}^{-1} \) with a density of 1.0109 g cm\(^{-3}\) at 23.8°C. The concentration of the stock solution is therefore 989.2 \( \mu \text{g g}^{-1} \). To calculate the uncertainty in the concentration of the dilute working standard, an estimate of the uncertainty in the concentration of the stock solution is required. The uncertainty in the concentration of the solution as specified by the supplier is assumed to be a rectangular distribution. The standard uncertainty, \( u(C_{\text{stock(w/v)}}) \), is therefore obtained by dividing the stated uncertainty by \( \sqrt{3} \), which gives 1.73 \( \mu \text{g ml}^{-1} \) [3,4]. There is also an uncertainty associated with the value of the density used in the conversion. The uncertainty in the density quoted by the supplier is estimated as ±0.0001 g cm\(^{-3}\) (i.e. the uncertainty in the last decimal place quoted). If a rectangular distribution is assumed, this gives 0.000058 g cm\(^{-3}\) as a standard uncertainty. The temperature in our laboratory is controlled at 23 ± 3°C with 95% confidence. Based on observations for water [17], the rate of change of density with temperature across this temperature range is estimated as 0.0002 g cm\(^{-3}\) °C\(^{-1}\). The uncertainty in the density value due to temperature variations is therefore \( (0.0002 \times 3)/1.96 = 0.00031 \text{ g cm}^{-3} \). The total uncertainty associated with the value of the density used, \( u(d) \), is therefore:

\[
u(d) = \sqrt{0.0000058^2 + 0.00031^2} \equiv 0.00032 \text{ g cm}^{-3} \tag{11}
\]

Combining these contributions gives the uncertainty in the concentration of the stock solution on
a w/w basis:

$$u(C_{\text{stock(w/w)}}) = 989.2 \times \sqrt{\left(\frac{1.732}{1000}\right)^2 + \left(\frac{0.00032}{1.0109}\right)^2}$$

$$\equiv 1.74 \mu g \text{ g}^{-1}$$  \hspace{1cm} (12)

The concentration of the dilute working standard, $C_{\text{dil}}$, is given by:

$$C_{\text{dil}} = \frac{C_{\text{stock(w/w)}} \times W_{\text{stock}}}{W_{\text{final}}}$$  \hspace{1cm} (13)

where $W_{\text{stock}}$ is the weight of the stock solution taken and $W_{\text{final}}$ is the final weight of the dilute working standard. As in the case of the dilution factor, only the precision associated with $W_{\text{stock}}$ and $W_{\text{final}}$ have been included in the uncertainty budget. Replicate weighings of 1 and 100 g calibrated weights had standard deviations of 0.00006 and 0.00024 g, respectively. Combining these values with the uncertainty calculated for the concentration of the stock solution gives an uncertainty in the concentration of the dilute working standard, $u(C_{\text{dil}})$, of:

$$u(C_{\text{dil}}) \equiv 9.98 \times \sqrt{\left(\frac{1.74}{989.2}\right)^2 + \left(\frac{0.00006}{1}\right)^2 + \left(\frac{0.00024}{100}\right)^2}$$

$$= 0.0174 \mu g \text{ g}^{-1}$$  \hspace{1cm} (14)

### 3.3. Calculation of standard and expanded uncertainty

The measurement uncertainty was calculated from the data given in Table 3. The combined standard uncertainty is calculated from the root sum of squares of the individual components, according to the rules set out in the Eurachem guide [3]. For samples containing at least 10 ng g$^{-1}$ $^{60}$Ni, the relative standard uncertainty was calculated as 0.0750 and for samples containing 3 ng g$^{-1}$ $^{60}$Ni as 0.188. Expanded uncertainties were calculated using a coverage factor of two which gives a level of confidence of \(95\%\). For samples containing 3 ng g$^{-1}$, the expanded uncertainty was calculated as 1.1 and as 1.5 ng g$^{-1}$ for samples containing 10 ng g$^{-1}$ $^{60}$Ni.

### 4. Conclusions

This study illustrates the application of cause-and-effect analysis to uncertainty estimation in analytical chemistry; in this case the determination of nickel in aqueous solutions by ICP-MS. We have found this to be a valuable tool for producing structured lists of uncertainty contributions. In this example, the major contributions to the uncertainty budget were found to be the method precision and the instrument drift (see Fig. 2). The contributions from the uncertainties associated with diluting the sample and the concentration of the dilute working standard were insignificant. The uncertainty for the method was calculated using a combination of existing QC data (results from the
analysis of a QC sample, precision data for the analytical balance), other available information (supplier’s information on the nickel stock solution used to prepare the calibration standards), and additional experiments planned to cover the parameters not covered by the existing information (in particular the method recovery).

The main advantage of using method performance parameters such as recovery and precision is that it allows a number of sources of uncertainty to be considered simultaneously, thus removing the need for time-consuming in-depth study of individual stages in the method. In addition, such information is often readily available in the form of validation studies or QC data, so minimising the amount of laboratory study required to complete the uncertainty budget. The main disadvantage is that grouping uncertainty components and considering them as single parameters, such as recovery, can make it difficult to identify exactly where the major sources of uncertainty arise. Such information is useful if the magnitude of the uncertainty estimate indicates that further method optimisation is required. However, the detailed list of influence parameters obtained during the cause-and-effect analysis can provide a useful starting point for planning additional optimisation work.

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References